A Theory of Wage and Turnover Dynamics*

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Abstract

The paper presents a theory of compensation – built on search and matching, firm-specific human capital, and self-enforcing wage contracts – that provides an unified explanation for a broad range of empirical observations on wage and turnover dynamics. For example, the model resolves the apparent puzzle posed by the lack of evidence of wage growth heterogeneity among jobs despite the fact that the same data show past wage growth on the job reduces turnover. The key implications of the model are as follows. First, wages increase and turnover rates decrease over the duration of an employment relationship, but the positive tenure effect on wages is predicted to be quantitatively weaker than the negative tenure effect on turnover. Second, within-job wage growth is higher and turnover is lower in high productivity growth jobs than in low productivity growth jobs. Third, serial correlation of within-job wage increases is indeterminate in spite of the assumed serial correlation of the underlying productivity increases on the job.

Key words: Firm-specific human capital, Search-and-matching, Wage renegotiation, Wage dynamics, Turnover.

JEL Classification: J30, J60.

1 Introduction

How wages are determined and why people move from one employment setting to another employment setting are classic questions in economics. More recent answers are based on a variety of considerations, including human capital investments, search-and-matching, incomplete information, learning, and incentives. A common feature among these compensation theories is the dynamic nature of wages and turnover. How wages are determined over the individual life cycle and employment duration, and the role of turnover in allocating workers among potential employers have taken center stage for the past several decades since the birth of modern labor economics. However, none of the extant theories offers a compelling account of the various and often conflicting findings on wage and turnover dynamics documented in the empirical literature. The motivation for this paper is to present a theory that can provide a unified explanation for a wide range of these empirical observations. In particular, the model of the paper addresses the following questions. Why do wages increase over the duration of an employment relationship? Why are worker-firm separations less likely as the employment relationship ages? Why is this latter negative tenure effect on turnover strong while the positive tenure effect on wages weak? Why does the negative tenure effect on turnover persists even when wages are held constant? Why does wage growth on a job reduce turnover? Why is serial correlation of within-job wage increases an inconclusive test of permanent differences in the rates of wage growth among jobs?

The theory builds on search-and-matching, firm-specific human capital, and self-enforcing wage contracts. The matching technology extends Jovanovic (1979a and 1979b) in the sense that each worker-firm pair is characterized by an idiosyncratic *productivity profile*. Put differently, a match is defined not only by a productivity level, as in the matching literature, but also by a match-specific productivity growth rate. This growth rate is the firm-specific human capital feature of the model. Heterogeneity of productivity profiles across all worker-firm pairs underpins a non-degenerate distribution of potential firms in the labor market. The search process of the model arises because workers have imperfect information about the location of the "best" match, and hence workers search for better alternatives while they are employed. The model retains the salient characteristic of search theory, namely, the optimal assignment of workers to firms in the presence of search frictions.

The next question is how are wages determined over the duration of an employment relationship? Since productivity increases on the job are firm-specific, time consistency dictates that firms do not have the ability to commit to future wage increases. Hence the wage setting mechanism of the model builds on counteroffer matching (Mortensen 1978; Postel-Vinay and Robin 2002), where firms have the ability to commit to fixed wage contracts, and incumbent firms can respond to such outside wage offers with counteroffers. In particular, each worker samples an outside firm from the same distribution in every period, and at the time of contact the outside firm makes a take-it or leave-it fixed wage offer based on match quality. If the outside wage offer is lower than the current wage then the incumbent firm retains the worker without offering a new renegotiated wage contract. Therefore wages on a job are downwardly rigid. If the outside wage offer is higher than the "maximum-matching-wage" – i.e. the highest outside wage offer the incumbent firm is willing to match – then the worker quits and moves costlessly to the other firm. However, if the outside wage offer falls between the current wage and the maximum-matching-wage then the incumbent firm retains the worker by offering a renegotiated fixed wage contract that exactly matches the outside wage offer. This wage policy of counteroffer matching is time-consistent and self-enforcing, and it is the source of within-job wage increases.

The final question is how are outside wage offers determined? The paper assumes that every outside wage offer, conditional on the match-specific productivity profile, is such that the present value of expected profits for the firm making the offer is equal to zero. The zero-profit wage rule allows an explicit characterization of the equilibrium wage offer function, and the model generates efficient turnover. The stringency of the zero-profit assumption notwithstanding, many of the modeling results on wage and turnover dynamics are robust to alternative specifications of the outside wage offer. Moreover, other non-competitive wage rules such as Postel-Vinay and Robin (2002) are also clearly consistent with efficient turnover.

The key theoretical problem is the derivation of the zero-profit equilibrium wage function, on the basis of which every firm makes their wage offers. The proof of existence of equilibrium is by construction: a candidate function expressed in terms of the primitives of the model – i.e. the productivity profile of the match and the distribution of such profiles – is shown to be the zero-profit equilibrium wage. This function is also the solution to the firm's profit maximization problem because the zero-profit wage is the highest outside wage offer the firm will match in every future period, which of course increases with tenure due to productivity growth. The explicit characterization of the equilibrium wage function shows that outside wage offers include a premium that is over and above the initial level of productivity of the match. This premium is equivalent to the present expected value of productivity increases that the worker is unable to extract in the future because of the luck of the draw. Since firms cannot commit to future wage increases in line with productivity increases, they pay this equivalent as an up-front compensating wage premium.

To preview, the main theoretical implications of the model and the intuitions for these results are as follows. First, wages increase and turnover rates decrease over the duration of an employment relationship. However, the positive tenure effect on wages is predicted to be quantitatively weaker than the negative tenure effect on turnover. These asymmetric tenure effects are the consequence of the fact that wages and turnover are not exactly determined by the same stochastic processes. The decrease in the turnover rate is a direct function of the increase in the highest outside wage offer the firm is willing to match (maximum-matching-wage) from one period to the next since outside wage offers arrive from a stationary distribution. However, wage increases on the job are primarily governed by the independent sampling process of outside wage offers, and are not a direct function of the maximum-matching-wage. The latter only serves as an *upper bound* for a renegotiated wage contract. Moreover, since wage increases occur if and only if the outside wage offer is higher than the previous period wage and lower than the maximum-matching-wage, the expected within-job wage increase is smaller than the corresponding increase in the maximum-matching-wage.

Second, the mean within-job wage growth rate is higher and turnover rate is lower in high productivity growth jobs than in low productivity growth jobs. The reason is because a firm will match a higher outside wage offer for a worker in a high growth job than for a worker in a low growth job since high growth jobs generate more firm-specific rents. And because workers in both jobs sample outside wage offers from the same distribution, expected wage growth is higher and turnover is lower in high growth jobs than in low growth jobs.

Third, the model implies that within-job wage increases in adjacent time periods will be negatively correlated for a given productivity profile, whereas the same covariance without the conditioning on the growth rate, is indeterminate. This inconclusive result on serial correlation of wage increases holds in spite of the assumed serial correlation of productivity increases. Note that wage increases on the job are equal to the difference between the outside wage offer and previous period wage if the outside wage offer is higher than the previous period wage and lower than the maximum-matching-wage. If the worker receives a high outside wage offer that raises within-job wages substantially then the likelihood of receiving an even higher wage offer in the next period is relatively low. Hence, conditional on a large wage increase, the expected wage increase in the next period is small. Conversely, if the worker receives a low outside wage offer that raises within-job wages only marginally (or not at all because the outside wage offer is less than or equal to the previous period wage) then the likelihood of receiving a wage offer in the next period that is higher than this low wage offer is relatively high. Hence, conditional on a small (or zero) wage increase, the expected wage increase in the next period is large. So the covariance of within-job wage increases in adjacent time periods is negative for any given productivity profile. However, this same covariance computed from a population of jobs with heterogeneous productivity growth rates has an ambiguous sign. Recall that the covariance of wage increases in adjacent time periods is the linear association of deviations of wage increases from their respective mean wage growth rates in adjacent time periods. With heterogeneous productivity growth rates the mean wage growth rates in adjacent time periods change, and hence the covariance without conditioning on the growth rate becomes indeterminate.

Although the model is designed to explain various findings on wage and turnover dynamics, some of the modeling results also raise doubts about the validity of widely used empirical procedures to estimate wage returns on tenure and experience. For example, Topel's two-step procedure (Topel 1991) is based on the presumption that within-job wage growth is an unbiased estimate of the joint returns on tenure and experience. However, if high wage growth jobs are more likely to be over sampled, as predicted by the model, then the first step of Topel's procedure is likely to lead to biased estimates of these joint returns on tenure and experience. More significantly, even when this selection problem is recognized as a potential source of bias, the lack of evidence of serial correlation of within-job wage growth is put forth as the reason for its empirical irrelevance (e.g., Topel 1991; Altonji and Williams 1997). But the model shows that serial correlation of wage growth is an inconclusive test of heterogeneity of permanent rates of wage growth among jobs. Moreover, since the model also predicts that wage growth is lower than the underlying productivity growth on the job, estimates of small wage returns on tenure should not be interpreted as specific skills playing a relatively minor role in human capital investments on the job. Initial wage offers, as argued in this paper, may include a premium for the future acquisition of specific skills.

The remainder of the paper is organized as follows. Section 2 presents various empirical findings and puzzles related to wage and turnover dynamics that no single extant theory can explain. Section 3 presents the basic model and derives the equilibrium wage function. This section concludes with an analysis of the properties of the equilibrium wage function and a critical assessment of the modeling assumptions. Section 4 derives several model implications that are consistent with the broad array of observations detailed in Section 2, and highlights some shortcomings of the paper. Section 5 on "Related Theory" clarifies how various features of the model relate to other theories of compensation and turnover, and especially to search-and-matching models. Section 6 concludes with a brief summary.

2 Empirical Findings

2.1 Tenure Effects on Wages and Turnover

Modern theories of compensation and turnover, ranging from firm specific human capital (Becker 1962) to Lazear type bonding models (Lazear 1981), are explicitly designed to show a positive relationship between wages and tenure and a negative relationship between turnover and tenure. The impetus for these earlier theoretical efforts is the widely documented empirical regularities of tenure effects on wages and turnover. Although the negative effect of tenure on turnover remains one of the most robust findings in empirical labor economics, the recent controversy about finding a positive tenure effect on wages has reignited a debate about the empirical importance of firm-specific skill investments.

Early empirical support for wage increases with job seniority was based on evidence of the positive cross-sectional association between seniority and earnings (e.g., Mincer and Jovanovic 1981). However, as Abraham and Farber (1987) and Altonji and Shakotko (1987) have argued, this evidence is insufficient to establish that earnings increase with seniority. For instance, if high wage jobs are more likely to survive than low wage jobs (say on account of heterogeneity of worker-firm match quality), then seniority will be positively correlated with high wages even though individual wages do not rise with seniority. Using longitudinal data and corrections for likely sources of heterogeneity bias, both these studies find that the cross-sectional return to tenure is largely a statistical artifact and that the true wage return to tenure is small if not negligible. In a later study Topel (1991) argues that wages do rise substantially with seniority. A subsequent reassessment by Altonji and Williams (1997) concludes that wage returns on tenure across all these different estimation procedures, though positive, are modest in size. Abowd et al. (1999) using a large longitudinal French data source also find that the estimated positive wage returns on tenure are small. On the other hand, Dustmann and Meghir (2005) using an administrative data set for Germany find more substantial returns on tenure for both skilled and unskilled workers.

The current consensus is that the positive wage returns on tenure are small despite the ubiquitous fact of a strong negative tenure effect on turnover. However, none of the existing workhorse theories can adequately explain this asymmetric tenure effect on wages and turnover. For example, bonding models (Lazear 1981) and selection models (Salop and Salop 1979) imply turnover decreases with tenure precisely because of back-loaded compensation designs. Matching models also directly couple turnover decreases to wage increases. Although Becker-type sharing models of specific capital investments imply wage increases that are smaller than the underlying productivity increases, the quit rate is a direct function of the worker's share of the costs and rewards in terms of higher future wages (Parsons 1972). Therefore, weak tenure effects on wages also imply weak tenure effects on quit rates. In a model of learning and specific skill accumulation, Felli and Harris (1996) argue that positive wage returns on tenure are a consequence of workers learning about their productivities in other firms while working in the current firm. However, in this model wage returns on tenure could be substantial and turnover is likely to increase with tenure. Hence these theories do not adequately address the observation of asymmetric tenure effects on wages and turnover.

By contrast, the model presented here implies not only the dual effects of tenure, like most of these other models, but more importantly, it also predicts asymmetric tenure effects on wages and turnover that is consistent with the facts.

2.2 Tenure Effects on Turnover holding Wages Constant

A related finding to the tenure effects discussed above is the negative multivariate relationship between tenure and turnover when the wage is held constant. For example, Topel and Ward (1992) find that turnover continues to decline with seniority despite holding the wage constant. This finding is troubling for matching models since they predict that the turnover rate will increase with tenure once the wage is held constant (Mortensen 1988; Galizzi and Lang 1998). The model in this paper, however, is consistent with this finding. Since wage renegotiation de-couples the wage from match value, the current wage does not necessarily reflect increases in match value with tenure. But since match value determines turnover the model predicts a negative tenure effect on turnover even when the wage is held constant.

2.3 Wage Growth, Turnover and Serial Correlation of Wage Growth

In the past two decades empirical studies using panel surveys of individual work histories and personnel records of large companies have analyzed within-job wage increases, persistence of wage growth, and correlations between wage growth and turnover. Bartel and Borjas (1981) find evidence of positive correlation between completed tenure and within-job wage growth. In a more conclusive study, Topel and Ward (1992) find that jobs offering higher wage growth are significantly less likely to end in worker-firm separations. This finding not only implies that the source of wage growth must have a firm specific component, but also heterogeneity of wage growth rates among jobs. However, two studies (Topel 1991; Topel and Ward 1992), based on the time series properties of within-job wage changes, conclude that heterogeneity in permanent rates of wage growth among jobs is empirically unimportant. So the direct evidence seems to suggest that jobs do not in fact differ in their prospects for wage growth. Note, the data used in the second study are the same data that show past wage growth reduces turnover, and hence the puzzle laid out in the abstract: lack of evidence of differences in wage growth rates despite the fact that the same data show past wage growth on a job reduces turnover.

Taken together, Topel and Ward's two findings – the negative correlation between wage growth and turnover, and the lack of evidence of serial correlation of wage growth – pose a challenge for the widely accepted "mismatch" theory of turnover (Jovanovic 1979a). Since the current wage is a sufficient statistic for job value, the mismatch theory is consistent with lack of evidence of positive serial correlation of wage growth. But the theory cannot explain the negative correlation between wage growth and turnover since it predicts that turnover should decline as a function of the wage level and not as a function of wage growth. On the other hand, assuming heterogeneity of wage growth rates can of course explain the negative correlation between wage growth and turnover (Munasinghe 2000), but such an assumption does not reckon well with the inconclusive evidence on wage growth persistence. One main objective of this paper is to explain jointly why past wage growth on a job reduces turnover and why within-job wage increases might be serially uncorrelated.

Other studies have presented evidence of positive serial correlation of wage increases. For example, Baker et al. (1994) using personnel records of managerial employees in a large firm find evidence of positive serial correlation of wage increases in adjacent time periods. In a related study, Abowd et al. (1999) show evidence of substantial variation in the estimated wage tenure slopes across firms despite the fact that the estimated wage return on tenure is small. The model in this paper is consistent with this gamut of findings since it implies precisely a relatively small but heterogeneous tenure effect on wages, and indeterminacy of serial correlation of within-job wage increases.

The mixed evidence of serial correlation of wage increases may also seem consistent with a class of wage models characterized by learning about worker ability and downward wage rigidity. For example, in Harris and Holmstrom (1982) rigid wage contracts are replaced by new wage contracts if the worker receives a better offer from the market. Chiappori et al. (1999) label such models as LDR models (for learning and downward rigidity), and derive a so-called "late-beginner property." This late-beginner property says that holding the current wage constant the future wage is negatively correlated with the past wage. As a result, the covariance of successive wage increases is positive.[1] This correlation, however, is likely to remain positive even without conditioning on the current wage due to what Chiappori et al. call the "fast-track" effect. Namely, that low (high) ability workers are likely to experience low (high) wage increases in successive periods. In contrast, the model here predicts that the covariance of successive wage increases is negative for a given productivity profile, whereas the same covariance without the conditioning is indeterminate.

2.4 Wage Adjustments and the Business Cycle

The model is also consistent with observed wage adjustments over the business cycle. For example, Beaudry and DiNardo (1991) find that current wages are negatively correlated

[1]Denote w_t as the wage at time t. The late-beginner property says that w_3 and w_1 are negatively correlated, holding w_2 constant. This implies that successive wage increases – i.e. $(w_3 - w_2)$ and $(w_2 - w_1)$ – are positively correlated conditional on w_2.

with the lowest realized unemployment rate since workers began with their present employer, whereas the current unemployment rate and the unemployment rate at the time of job start have a smaller impact. Wage renegotiation implies upward wage revision due to receipt of better outside wage offers, and clearly a low unemployment rate increases the likelihood of receiving better outside wage offers. Since employment relationships have rents over and above the current wage, it is not surprising that the lowest unemployment rate over the duration of employment has a significantly larger impact on current wages. During periods of high labor demand wages are more likely to be renegotiated up because of more wage offers, and during periods of less labor demand wages are more likely to remain sticky.

2.5 Establishment Level Wages and Quit Rates

Another noteworthy finding, based on an Italian data source, is that conditional on their own wage, workers in establishments that pay higher wages to similar workers are less likely to quit (Galizzi and Lang 1998). The authors claim that higher wages paid to similar workers should be interpreted as expected future wage growth. If so, this finding is consistent with the model since it predicts lower turnover among workers with higher wage growth prospects.

2.6 Other Related Findings

The search-and-matching and wage renegotiation features of the model also imply other widely documented observations on wages and turnover. For example, wage increases and turnover decreases over the individual life cycle are direct implications of models with search frictions. Workers with long labor market experience are more likely to have found jobs with higher wages, and hence less likely to turnover. These patterns are widely documented in the empirical literature. Moreover, evidence of positive wage gains among movers, especially among those who "quit" their jobs (Mincer 1986), is also consistent with the model. Note, since workers only quit when they receive a wage offer that is higher than the maximum-matching wage, the model predicts positive mobility wage gains conditional on a quit.

Matching models generate equilibrium wage dispersion.[2] Since matching with heterogeneous growth rates and wage renegotiation create more complex wage outcomes, the model here generates wage dispersion even if observed and unobserved worker and job characteristics such as tenure, experience, and match quality are held constant. The reason is because wage renegotiation creates an essential indeterminacy of current wages that must

[2] Mortensen (2003) addresses the question of wage dispersion from the theoretical framework of search-and-matching and presents supportive empirical evidence. Other recent search-and-matching models such as Moscarini (2005) and Shimer (2004) also address the question of wage dispersion by constructing models that are more amenable to estimation on the basis of well-known properties of empirical wage distributions.

lie between the initial equilibrium wage (which differs on account of match quality) and the maximum-matching-wage in future time periods (which differs on account of heterogeneity of productivity growth rates).[3]

A final implication of wage renegotiation is the cluster of observations at the point of zero nominal within-job wage changes. If an outside wage offer is less than or equal to the current wage then the next period wage remains unchanged. Hence the model predicts a cluster of exactly zero nominal wage changes. McLaughlin (1994) presents evidence of nominal wage changes of those staying with the same employer using the Panel Study of Income Dynamics (1976-1986), where the striking feature is indeed the large cluster of observations at precisely the point of zero nominal changes. Baker et al. (1994) also find among a sample of managerial employees a significant cluster at zero nominal salary increases.

3 Model

3.1 Assumptions

Firm-specific human capital, search-and-matching, and self-enforcing wage contracts are the three basic elements of the model. This section presents a formalization and description of each of these features of the model. The first assumption is the distribution of productivity profiles across all worker-firm pairs. Each worker-firm match is characterized by an initial productivity level and a growth rate that determines future productivity on the job. Productivity increases on the job are firm-specific and this skill accumulation occurs automatically at the match-specific growth rate. The production technology follows Jovanovic (1979a and 1979b): firm production functions exhibit constant returns to scale and labor is the only factor of production. Hence firm size is indeterminate. Each worker-firm pair therefore can be treated independently because each productivity profile is independent of firm size.

> ASSUMPTION 1. Workers face an infinite number of potential firms and each worker-firm match is characterized by a two-dimensional vector $\sigma \equiv (p, g)$, where p is the initial productivity level and $g > 1$ is the growth rate of productivity. Hence a worker in the t^{th} period of employment in a particular firm has productivity $g^t p$. Also $\sigma \in \Sigma \subset R^2_+$, where Σ is compact and ϕ is a nonatomic

[3] These implications are comparable to Burdett and Coles (2003) where there is both initial wage dispersion and wage increases with tenure. Workers and firms are homogeneous in their model in part because they want to explain wage dispersion across workers who are observationally equivalent. The model here of course assumes heterogeneity of match quality across worker-firm pairs, but workers are nonetheless observationally equivalent. For a recent survey of wage dispersion in search-theoretic models see Rogerson et al. (2004).

probability measure on Σ. Workers are infinitely lived and β is the common discount factor for both workers and firms. Furthermore $\underset{\sigma\in\Sigma}{Max}\; g(\sigma) < \frac{1}{\beta}$.

Various aspects of Assumption 1 need to be clarified. Following the matching literature, there are neither good nor bad workers or firms, but only good or bad matches. Hence each worker-firm productivity profile is strictly match-specific and all workers *ex ante* are identical.[4] Moreover, each worker faces the identical distribution ϕ of productivity profiles.[5] The standard assumption of matching models is a non-degenerate distribution of idiosyncratic productivity *levels* across all worker-firm pairs. This matching idea is extended here by including a match-specific productivity *growth* rate as a second, human capital dimension of a worker-firm match. Typically investments in firm-specific skills are endogenously determined by the worker-firm match quality (e.g. Jovanovic 1979b; Bartel and Borjas 1981) that implies a positive correlation between p and g since the growth rate g would be endogenously determined by the level of match quality p. Assumption 1 does not impose any *a priori* restriction because some of the model implications rest on a less strict correlation between p and g. (See the discussion in Section 4.4.)

A final observation is that the assumption of deterministic productivity profiles sacrifices some descriptive realism for analytical simplicity. Although there is empirical evidence that within-job wages evolve as a random walk with drift (Topel 1991), there is no such evidence on the evolution of within-job productivity. Since a deterministic productivity profile both simplifies the analysis and generates a rich set of implications for wage and turnover dynamics, a noise component is excluded from the characterization of a productivity profile.

The second assumption is the existence of search frictions in the labor market. Hence search for better jobs is costly and workers do not immediately find the best match. As a consequence workers search for better alternatives while they are employed.

ASSUMPTION 2. At the end of every period, a worker receives an outside job offer from a firm with match quality $\widetilde{\sigma}$ drawn randomly from Σ according to ϕ.

[4] Since productivity profiles are match-specific, the model implications provide a structural explanation of findings related to wage and turnover dynamics without appealing to worker or firm heterogeneity. Note, the various empirical studies cited in Section 2 have extensive controls for individual and firm level characteristics, including a host of human capital variables such as education and experience. In addition, these empirical analyses implement various econometric procedures to correct for unobserved individual fixed effects.

[5] This assumption would be immediate if productivity profiles are specific to firms. However, given that productivity profiles are specific to each worker-firm match, the assumption that every worker faces the same *distribution* of productivity profiles is more stringent. Note that match-specificity implies firm-specificity, but not the other way round. Hence the use of the term "firm-specific" refers to match specificity and not the fact that each firm has a specific productivity profile no matter who is employed at the firm.

This formulation implicitly treats jobs as "inspection goods" in the tradition of Burdett (1978) and Jovanovic (1979b). Put simply, the productivity profile is known at the time the worker receives the outside offer. Hence there is no "learning" about match quality as in Jovanovic (1979a) or "learning" about worker ability as in Harris and Holmstrom (1982). Since job offers are publicly observed it is a model with complete information. The salient feature is that search is costly and therefore the worker receives only a single (finite) job offer in every period. Search effort, however, is exogenous in the model as indicated by the constant offer arrival rate. The possible ramifications of endogenous search effort for the modeling results are discussed in Section 3.4.3 below.

The third assumption specifies the wage setting mechanism in the presence of search frictions and productivity growth. Since productivity increases are firm-specific there is no competition for such skills from outside firms. So firms do not set wages equal to productivity in every period. Firms increase wages only if the worker receives a better wage offer.

> ASSUMPTION 3. The outside job offer entails a zero-profit wage $\mathbf{w}(\sigma) : \Sigma \longrightarrow R^{+}$. If this outside wage offer is higher than the worker's current wage the incumbent firm can match this offer and retain the worker or allow the worker to costlessly move to the other firm. Moreover, firms are not allowed to renege on renegotiated wage contracts and hence wages remain constant until such time as a worker receives from another firm an offer of a higher wage.

Although firms are unable to commit to future wage increases, the initial fixed wage offer[6] is assumed to be a zero-profit wage – i.e. the wage function $\mathbf{w}(\sigma)$ is such that the present value of profits over the expected duration of employment is equal to zero. Hence lifetime rents due to the luck of the draw go to the worker. This assumption is key to the explicit derivation of the equilibrium wage function. The stringency of this zero-profit wage assumption and robustness of the modeling results to alternative specifications of the outside wage offer are discussed in detail in Section 3.4.2 below.

Since firms increase wages if and only if the worker receives a better outside wage offer, the single period payoffs to the worker and firm of this renegotiation policy are given as follows. Suppose at time t the worker receives a wage w_t and produces $g^t p$, where p is productivity at the time of job start. At time period $t+1$ the worker receives $max\{w_t, \mathbf{w}(\widetilde{\sigma})\}$ and produces

[6]Both Stevens (2004), and Burdett and Coles (2003) consider on-the-job search and matching models where firms post more complicated wage-tenure contracts compared to the standard wage posting model of Burdett and Mortensen (1998). Stevens shows that with risk neutral workers a step wage-tenure contract is optimal, and in equilibrium all firms offer the same step contract and there is no turnover. By contrast, Burdett and Coles show that if workers are strictly risk averse then in equilibrium firms offer different wage-tenure contracts, and hence their model also generates turnover.

either $g^{t+1}p$ if the worker remains with the incumbent firm or $p(\widetilde{\sigma})$ if the worker quits and moves to the new firm with match quality $\widetilde{\sigma}$. The profit for the incumbent firm at time t is $g^t p - w_t$, and the profit at time period $t+1$ is $g^{t+1}p - max\{w_t, \mathbf{w}(\widetilde{\sigma})\}$ if the firm keeps the worker, and 0 if the worker quits. The profit for the new firm at time period $t+1$ is $p(\widetilde{\sigma}) - \mathbf{w}(\widetilde{\sigma})$ if the worker quits the incumbent firm and joins the new firm, and 0 otherwise.[7]

Downward wage rigidity of the model is due to the presumption of legal restrictions that prevent firms from reneging on renegotiated wage contracts (see Postel-Vinay and Robin, 2002).[8] In the literature, various theoretical considerations have been expounded that lead to downward wage rigidity. For example, in Harris and Holmstrom (1982) downward wage rigidity acts as an insurance policy for workers where the economic environment is characterized by productivity risks due to learning about worker ability, and because the employer is risk neutral and the worker is risk averse. MacLeod and Malcomson (1993) show that downward wage rigidity can induce efficient investments in some circumstances of the holdup problem. Empirical evidence shows that nominal wages are indeed downwardly rigid, although real wage cuts are not uncommon (Baker et al. 1994). In a companion paper, Munasinghe and O'Flaherty (2005) generate real wage cuts by excluding *ex post* offer matching within a similar theoretical framework.

In the model presented here the impetus for within-job wage growth is both the receipt of better outside wage offers and the wage renegotiation policy. From the worker's perspective the source of any – i.e. within-job or between-job – wage increase is the receipt of a better outside wage offer. As a consequence, the wage at any given time is a sufficient statistic for the job value to the worker as shown below. Moreover, since this wage setting mechanism is self-enforcing, it does not rely on reputation repercussions to be enforced like the matching models of Jovanovic (1979a and 1979b).

A final observation is the absence of wage and turnover dynamics in the long-run due to the infinite-horizon feature of the model. Since workers live forever, all workers eventually end up in jobs with wages that match the highest outside wage offer, which implies the absence of both within-job wage increases and turnover in the long-run. Hence the model does not generate any implications for the steady-state distribution of σ across on-going matches. Although this issue could be addressed for example by assuming exogenous entry and exit rates of workers into and out of the labor market, the model here is not designed to study steady-state distributions of σ, and its ramifications on wage and turnover dynamics.

[7] The model excludes mobility costs. Mobility costs can also create a wedge between current and outside job values, but firm-specific productivity growth generates richer wage and turnover dynamics.

[8] In this model a firm's lack of "commitment ability" only implies that it cannot credibly promise to increase wages in the future because productivity increases are firm specific. But that does not necessarily preclude firms from cutting wages.

Moreover, the implication that wages in the long run converge to the highest outside wage offer is counter-factual. Note, however, that the model assumes all workers are identical.

3.2 Existence of the Equilibrium Wage Function

Given assumptions 1 through 3 the worker's only decision is to quit and join the outside firm with match quality $\widetilde{\sigma}$ if the outside wage offer $\mathbf{w}(\widetilde{\sigma})$ is greater than the current wage and the incumbent firm does not match this outside wage offer.[9] The firm's problem is two-fold: first, it must make a fixed wage offer to a new worker, and second, it must determine the highest outside wage offers it will match in all future time periods. The profits of any given firm depends on the wage policies chosen by other firms since the latter affect the distribution of wage offers and duration of the employment relationship. The existence of an equilibrium wage function $\mathbf{w}$ is addressed next.

For a given ϕ and function $\mathbf{w}$, the CDF of outside wage offers is determined. Let $F(w', \mathbf{w}) = \phi(\sigma \mid \mathbf{w}(\sigma) \leq w')$ denote this CDF given $\mathbf{w}$. Hence $F(w', \mathbf{w})$ is the probability of getting an offer at most w', given ϕ and $\mathbf{w}$. Then the present expected value of profits to the firm from a worker with match quality σ and a current wage w is given by:

$$\Pi(\sigma, w, \mathbf{w}) = p - w + \beta \max_{w_1} \left\{ \Pi(g\sigma, w, \mathbf{w}) F(w, \mathbf{w}) + \int_{w}^{w_1} \Pi(g\sigma, w', \mathbf{w}) dF(w', \mathbf{w}) \right\},$$

where $g\sigma \equiv (gp, g)$ – i.e. the productivity profile starting in the next period – and w_1 is the highest outside wage offer the firm will match in the next period. So the present value of profits is equal to current profits $p - w$, plus the present value of expected profits in the next period (given by the sum of the two terms within the bracket). The first term inside the bracket is expected profits in the next period if the outside wage offer is less than the current wage w since the wage for the next period then remains unchanged.[10] The second term is expected profits in the next period if the outside wage offer w' falls between the current wage w and the highest outside wage offer w_1 the firm is willing to match since the next period wage is then equal to the outside wage offer. Note, this is the region of job offers where the firm matches the outside wage offer. If the outside wage offer w' is greater than w_1 then the firm does not match the outside wage offer, and the worker quits and moves to the other firm. Note further, when a specific employment relationship terminates the firm no longer makes any profits from that worker, but the firm continues to exist. Since the production

[9] Although this decision depends on the wage offers, the model here, like Harris and Holmstrom (1982), does not give the worker a major role in terms of individual choice.

[10] The firm cannot reduce next period wages in the event that a new outside wage offer is less than the wage the firm is currently paying since it is illegal to renege on renegotiated wage contracts.

technology is constant returns to scale and firm size is indeterminate, every employment relationship can be treated independently.

Profit maximization implies that the firm will set the highest outside wage offer it matches w_1 to satisfy a zero expected-profit condition: $\Pi(g\sigma, w_1, \mathbf{w}) = 0$. If w_1 implies positive or negative expected profits then the firm is clearly not maximizing profits. For example, if w_1 is such that $\Pi(g\sigma, w_1, \mathbf{w}) > 0$ then the firm will allow a worker to quit for some outside wage offers greater than w_1 in spite of the fact that a counteroffer would retain the worker and yield some positive expected profits. Conversely, if $\Pi(g\sigma, w_1, \mathbf{w}) < 0$ then the firm will match some outside offers that would retain the worker, but yield negative expected profits.

The zero-profit wage rule also implies that the initial wage offer w must be such that: $\Pi(\sigma, w, \mathbf{w}) = 0$. Hence the equilibrium wage function $\mathbf{w} : \Sigma \longrightarrow R^+$ must satisfy

$$\Pi(\sigma, \mathbf{w}(\sigma), \mathbf{w}) = 0, \text{ for all } \sigma.$$

If $\mathbf{w}(\cdot)$ is the equilibrium wage function then the initial zero-profit equilibrium wage $\mathbf{w}(\sigma)$ is the actual wage paid to the worker at the start of employment, and the subsequent zero-profit wages over the duration of the employment relationship, which is also given by the same function – i.e. $\mathbf{w}(g^t\sigma)$, $\forall t > 0$, where $g^t\sigma \equiv (g^t p, g)$ – are the highest outside wage offers the firm will match in the future. Hence the equilibrium wage function $\mathbf{w}(\cdot)$ that satisfies $\Pi(\sigma, \mathbf{w}(\sigma), \mathbf{w}) = 0$ is the solution to the firm's two-fold problem.

The proof of existence of the equilibrium wage function $\mathbf{w}$ is by constructing a function that satisfies the condition $\Pi(\sigma, \mathbf{w}(\sigma), \mathbf{w}) = 0$, for all σ. First, denote $W(\sigma)$ as "match value," and define it as the highest present value of expected lifetime productivity of a worker with match quality σ:

$$\begin{aligned} W(\sigma) &= p + \beta E \max \{W(g\sigma), W(\widetilde{\sigma})\} \\ &= p + \beta \left\{ W(g\sigma)\phi(\widetilde{\sigma} | W(\widetilde{\sigma}) \leq W(g\sigma)) + \int_{\{\widetilde{\sigma} | W(\widetilde{\sigma}) > W(g\sigma)\}} W(\widetilde{\sigma}) d\phi(\widetilde{\sigma}) \right\}. \end{aligned}$$

The first result is given below.

LEMMA 1. $W(\sigma)$ exists and it is increasing in p and g.

PROOF. See Appendix.

Match value is the present value of lifetime productivity under a policy of optimal turnover, and this match value increases with tenure because it is an increasing function of the productivity level p and growth rate g. The match value function also represents the social planner's solution to the problem characterized by Assumptions 1 and 2. The next proposition claims that equilibrium wage of the market mechanism described in Assumption 3 is a function of current productivity and difference in future and current match values.

PROPOSITION 1. Given Assumptions 1 to 3, for every $\sigma \in \Sigma$ and a given ϕ,

$$\mathbf{w}(\sigma) = p + \beta \left\{ \begin{array}{l} (W(g\sigma) - W(\sigma))\phi(\widetilde{\sigma}|W(\widetilde{\sigma}) \leq W(\sigma)) \\ + \int_{\{\widetilde{\sigma}|W(\sigma)<W(\widetilde{\sigma})\leq W(g\sigma)\}} (W(g\sigma) - W(\widetilde{\sigma}))d\phi(\widetilde{\sigma}) \end{array} \right\} \tag{1}$$

is the zero-profit equilibrium wage function.

The proof of Proposition 1 is based on several lemmas.

LEMMA 2. $\mathbf{w}(\sigma)$ is a monotone transformation of $W(\sigma)$.

PROOF. See Appendix.

Lemma 2 says that if any two jobs have the same match value then the equilibrium wage offer will be the same for both jobs, and that if one job has a higher match value than another job then the equilibrium wage offer will be higher for the first job than for the second job.

In order to prove that this candidate function is the fixed point solution – i.e. the equilibrium wage function – to the firm's problem given by $\Pi(\sigma, \mathbf{w}(\sigma), \mathbf{w}) = 0$, we need to characterize the present value of wage payments to a worker under a policy of wage renegotiation. Denote $V(w, \mathbf{w})$ as "job value," and define it as the present value of expected lifetime wage payments to a worker when the incumbent firm pays a wage w and the worker receives a single outside wage offer in every period given by $\mathbf{w}(\widetilde{\sigma})$. Recall, for any $\mathbf{w}$ function the CDF of wage offers is given by $F(w, \mathbf{w}) = \phi(\widetilde{\sigma}|\mathbf{w}(\widetilde{\sigma}) \leq w)$. Hence, given a current wage w and some function $\mathbf{w}$, job value is expressed as follows:

$$\begin{aligned} V(w, \mathbf{w}) &= w + \beta E \max\{V(w, \mathbf{w}), V(\mathbf{w}(\widetilde{\sigma}), \mathbf{w})\} \\ &= w + \beta \left\{ V(w, \mathbf{w})\phi(\widetilde{\sigma}|\mathbf{w}(\widetilde{\sigma}) \leq w) + \int_{\{\widetilde{\sigma}|\mathbf{w}(\widetilde{\sigma})\geq w\}} V(\mathbf{w}(\widetilde{\sigma}), \mathbf{w})d\phi(\widetilde{\sigma}) \right\} \end{aligned}$$

The only source of wage increase for the worker is the receipt of a better outside wage offer. Job value, unlike match value, is independent of the turnover rule since the worker is indifferent about whether a wage increase occurs because the incumbent firm matches an outside offer or because the worker moves to another firm. Hence, for any given $\mathbf{w}$, job value is only a function of the current wage w and it is a monotonically increasing function of w.

Given the definitions of match value and job value, the next lemma states that the candidate zero-profit equilibrium wage function $\mathbf{w}(\sigma)$ given in equation (1) is constructed by setting job value equal to match value.

LEMMA 3. The function $\mathbf{w}(\sigma)$ that solves $V(\mathbf{w}(\sigma), \mathbf{w}) = W(\sigma)$ is given by the candidate equilibrium function in (1).

PROOF. Since $\mathbf{w}(\sigma)$ is a monotone transformation of $W(\sigma)$ (from Lemma 2) we can write the difference between match value and job value as follows:

$$W(\sigma) - V(\mathbf{w}(\sigma), \mathbf{w}) = p - \mathbf{w}(\sigma) + \beta \left\{ \begin{array}{l} (W(g\sigma) - V(\mathbf{w}(\sigma), \mathbf{w}))\phi(\widetilde{\sigma}|W(\widetilde{\sigma}) \leq W(\sigma)) \\ + \int_{\{\widetilde{\sigma}|W(\sigma)<W(\widetilde{\sigma})\leq W(g\sigma)\}} (W(g\sigma) - V(\mathbf{w}(\widetilde{\sigma}), \mathbf{w}) d\phi(\widetilde{\sigma}) \\ + \int_{\{\widetilde{\sigma}|W(\widetilde{\sigma})>W(g\sigma)\}} (W(\widetilde{\sigma}) - V(\mathbf{w}(\widetilde{\sigma}), \mathbf{w}) d\phi(\widetilde{\sigma}) \end{array} \right\}.$$

Since $W(\sigma) = V(\mathbf{w}(\sigma), \mathbf{w})$ for all σ, by substitution we get:

$$0 = p - \mathbf{w}(\sigma) + \beta \left\{ \begin{array}{l} (W(g\sigma) - W(\sigma))\phi(\widetilde{\sigma}|W(\widetilde{\sigma}) \leq W(\sigma)) \\ + \int_{\{\widetilde{\sigma}|W(\sigma)<W(\widetilde{\sigma})\leq W(g\sigma)\}} (W(g\sigma) - W(\widetilde{\sigma}) d\phi(\widetilde{\sigma}) \\ + \int_{\{\widetilde{\sigma}|W(\widetilde{\sigma})>W(g\sigma)\}} (W(\widetilde{\sigma}) - W(\widetilde{\sigma}) d\phi(\widetilde{\sigma}) \end{array} \right\}.$$

Since the last term within the bracket drops out, $\mathbf{w}(\sigma)$ can be expressed as:

$$\mathbf{w}(\sigma) = p + \beta \left\{ \begin{array}{l} (W(g\sigma) - W(\sigma))\phi(\widetilde{\sigma}|W(\widetilde{\sigma}) \leq W(\sigma)) \\ + \int_{\{\widetilde{\sigma}|W(\sigma)<W(\widetilde{\sigma})\leq W(g\sigma)\}} (W(g\sigma) - W(\widetilde{\sigma}) d\phi(\widetilde{\sigma}) \end{array} \right\},$$

where this function is precisely the same as the candidate equilibrium wage function in (1) above.■

The final step of the proof of Proposition 1 is to show that a firm's profits are equal to the difference between match value and job value.

LEMMA 4. A firm's present value of expected profits from a worker with match quality σ and current wage w is equal to the difference between match value and job value: $\Pi(\sigma, w, \mathbf{w}) = W(\sigma) - V(w, \mathbf{w})$.

PROOF. Since $W(\sigma)$ is the present value of expected lifetime productivity of a worker and $V(w, \mathbf{w})$ is the present value of expected lifetime wage payments to a worker, $W(\sigma) - V(w, \mathbf{w})$ is the present value of expected lifetime profits. These aggregate profits are of course distributed across the current firm and other outside firms that the worker might move to in the future. But whenever a worker moves to another firm, say with match quality σ', the equilibrium wage offer $\mathbf{w}(\sigma')$ is such that $W(\sigma') - V(\mathbf{w}(\sigma'), \mathbf{w}) = 0$ (from Lemma 3), which implies that aggregate expected profits at the time a worker begins employment at any new firm is equal to zero. Hence the present value of expected lifetime profits due to job changes in the future is equal to zero. Since $W(\sigma) - V(w, \mathbf{w})$ is the discounted sum of expected profits in the current firm and in all outside firms the worker might move to in the future, and because the latter profits equal zero, the expected profits of the firm $\Pi(\sigma, w, \mathbf{w}) = W(\sigma) - V(w, \mathbf{w})$, which completes the proof of Lemma 4. Moreover, since $W(\sigma) - V(\mathbf{w}(\sigma), \mathbf{w}) = 0$ (from Lemma 3), $\mathbf{w}(\cdot)$ is the solution to the fixed point problem: $\Pi(\sigma, \mathbf{w}(\sigma), \mathbf{w}) = 0$, for all σ, which then completes the proof of Proposition 1.■

Hence $\mathbf{w}(\sigma)$ given in (1) solves the firm's two-fold problem. For any given σ, a firm will make a wage offer $\mathbf{w}(\sigma)$ and will match any outside wage offer no larger than $\mathbf{w}(g^t\sigma)$ in every future period t. Since every firm uses this same function to make wage offers, $\mathbf{w}(\sigma)$ is the zero-profit equilibrium wage function. So for any exogenously given distribution of productivity profiles ϕ, the equilibrium wage offer distribution $\mathbf{w}(\widetilde{\sigma})$ is endogenously determined by model. The next subsection analyses the properties of this equilibrium wage function.

3.3 Properties of the Equilibrium Wage Function

Since every outside wage offer $\mathbf{w}(\sigma)$ is such that $W(\sigma) = V(\mathbf{w}(\sigma))$ for all σ, and $F(w) = \phi(\widetilde{\sigma}|\mathbf{w}(\widetilde{\sigma}) \leq w)$, the equilibrium wage function can be re-written as:[11]

$$\mathbf{w}(\sigma) = p + \beta \left\{ (W(g\sigma) - V(\mathbf{w}(\sigma)))F(\mathbf{w}(\sigma)) + \int_{\mathbf{w}(\sigma)}^{\mathbf{w}(g\sigma)} (W(g\sigma) - V(\mathbf{w}(\widetilde{\sigma})))dF(\mathbf{w}(\widetilde{\sigma})) \right\}.$$

Under this formulation, the interpretation of the wage premium given by the discounted sum of the terms within the bracket above is straightforward. First note that next period profits are given by $W(g\sigma) - V(w')$ if the wage in the next period is w'. The first term inside the bracket is expected profits in the next period if the outside wage offer is less than the current equilibrium wage $\mathbf{w}(\sigma)$ – i.e. if the outside wage offer falls in the region where the wage remains unchanged. The second term is expected profits in the next period if the outside wage offer falls between $\mathbf{w}(\sigma)$ and the highest outside wage offer the firm is willing to match $\mathbf{w}(g\sigma)$ – i.e. if the outside wage offer falls in the wage renegotiation region where the firm matches the outside wage offer. Hence the sum of these two terms is the expected profits the firm extracts in the next period because it does not increase wages unless the worker receives a better outside wage offer. Since the equilibrium wage $\mathbf{w}(\sigma)$ is a zero-profit wage, these expected future profits that the worker is unable to collect due to the luck of the draw are included as an up-front payment. Put differently, because firms are unable to commit to

[11] For notational brevity, from now on the second argument $\mathbf{w}$ in the definitions of V and F are dropped.

future wage increases they must include this compensating up-front wage premium in their zero-profit equilibrium wage offers.

This equilibrium wage function also generates optimal turnover since the worker switches jobs only if the outside offer has a higher match value than the current match value.

COROLLARY 1. $\mathbf{w}(\cdot)$ generates optimal turnover.

PROOF. Profit maximization implies that a firm will allow a worker to quit a job in the next period if and only if $\mathbf{w}(g\sigma) < \mathbf{w}(\widetilde{\sigma})$. Since $\mathbf{w}(\cdot)$ is a monotone transformation of $W(\cdot)$ (from Lemma 2) it follows that the worker will quit a job if and only if $W(g\sigma) < W(\widetilde{\sigma})$ – that is, if the match value of the outside job offer is greater than the match value in the incumbent firm. Since turnover is optimal in the definition of match value, $\mathbf{w}(\cdot)$ also generates optimal turnover.■

This result shows that the wage renegotiation policy generates optimal turnover and thus mimics the social planner's solution to the allocation of workers among jobs given search frictions. The efficiency of this market mechanism is due not only to the assumption that equilibrium wage offers are zero-profit, but also because search effort is exogenous.[12]

Since match value is increasing in p and g, and job value is increasing in wages, the equilibrium wage function $\mathbf{w}(\cdot)$ is clearly increasing in p and g. Moreover, the wage premium $(\mathbf{w}(\sigma) - p)$ is a function of g and match value $W(\sigma)$. A high growth rate implies a high wage premium because future productivity is higher and not all of it can be captured by the worker in the future. For instance, in the absence of productivity growth there is no wage premium since $W(g\sigma) = W(\sigma)$ and hence $\mathbf{w}(\sigma) = p$. Also note that the wage premium is positively related to match value. For example, consider a job with the highest match value $\overline{\sigma}$, i.e., $W(\widetilde{\sigma}) \leq W(\overline{\sigma})$, $\forall\widetilde{\sigma}$. Then the wage premium for$\overline{\sigma}$ is given by: $\mathbf{w}(\overline{\sigma}) - p(\overline{\sigma}) = \beta[W(g\overline{\sigma}) - V(\mathbf{w}(\overline{\sigma}), \mathbf{w})]$. The renegotiation term drops out because $\mathbf{w}(\overline{\sigma})$ is the highest outside wage offer. Hence the entire increase in match value must be collected up-front, which implies a relatively high wage premium. Conversely, if $\underline{\sigma}$ is the job with the lowest match value then the wage premium is given by:

$$\mathbf{w}(\underline{\sigma}) - p(\underline{\sigma}) = \beta \int_{\{\widetilde{\sigma}|W(\underline{\sigma})<W(\widetilde{\sigma})\leq W(g\underline{\sigma})\}} (W(g\underline{\sigma}) - V(\mathbf{w}(\widetilde{\sigma}), \mathbf{w}))d\phi(\widetilde{\sigma}).$$

Note that every wage offer in the next period that does not also result in turnover allows the worker to capture some (if $W(g\underline{\sigma}) > W(\widetilde{\sigma})$) or all (if $W(g\underline{\sigma}) \leq W(\widetilde{\sigma})$) of the increase in future match value, which implies a relatively low wage premium. The point is the wage

[12] See Section 3.4.3 for a more detailed discussion of exogenous search effort.

premium increases with match value because the likelihood of receiving a better wage offer decreases with match value, and hence the worker is less able to extract rents in the future.

Although wage offers yield zero-profits, for an identical σ the $\mathbf{w}(\sigma)$ could be very different depending on the distribution $\phi(\sigma)$. For instance, if the distribution of productivity profiles changes so that the likelihood of a better offer for a given σ increases then the wage premium will be lower since the worker can now extract more future rents given the more favorable distribution of outside wage offers. Clearly, the equilibrium wage depends on both the match-specific productivity profile σ and the probability distribution ϕ over Σ.

The explicit formulation of the equilibrium wage function also makes it transparent why the model here is both an external and internal labor market theory of wages. The equilibrium wage is clearly a function of both outside wage offers and productivity increases on the job. Put differently, within-job wage increases and turnover are the result of the interplay between "external" wage offers and "internal" productivity growth on the job.

3.4 Assumptions and Robustness of the Model

3.4.1 Heterogeneity of Firm-Specific Productivity Profiles

Before proceeding to the model implications, this section concludes with a critical assessment of the modeling assumptions. The basic presumption of the model is the existence of a non-degenerate distribution of firm-specific productivity profiles across all worker-firm pairs. This particular rendition of job matching as a "firm" specific phenomenon is essential to generate model implications consistent with the employment dimension of the empirical findings – i.e. within-*firm* wage dynamics and inter-*firm* labor mobility – detailed in Section 2. However, the critical assumption is not whether the firm is the appropriate demarcation of skill specificity *per se* because the applicability of this theoretical framework depends only on whether there are any employment dimensions – from industry classifications to specialized tasks – along which skill acquisition may be specific in the sense elaborated here. Some evidence suggests that skill acquisition on the job may be more industry specific than firm specific (Neal 1995). Other theoretical work (Gibbons and Waldman 2003) swings the pendulum in the opposite direction and introduces "task-specific" human capital to explain some features of internal labor markets. In principle, the model could be adapted to address industry or task specific skill acquisition and generate implications related industry or internal labor market compensation and mobility dynamics.

The second, and perhaps novel, aspect of a worker-firm match is that different work environments offer different opportunities for skill accumulation or on-the-job productivity growth. Empirical evidence of differences in productivity growth on the job is of course scarce, even though there is overwhelming evidence of differences in the provision of formal and informal training.[13] However, the idea that different jobs or work activities or occupations offer different learning and growth opportunities (e.g. Rosen 1972; Weiss 1971) is certainly not new in the labor theory. Also, heterogeneity of skill accumulation is the cornerstone of human capital theory as an explanation of personal income distribution (Mincer 1993). Although the model here generates a variety of basic results on wage and turnover dynamics holding productivity growth constant, heterogeneity of growth rates is essential for deriving implications on wage growth and turnover, and serial correlation of wage increases.

3.4.2 Competitive Wage Offers

The assumption that outside firms offer a zero-profit wage simplifies the derivation of the equilibrium wage function and allows an explicit characterization of this wage function. But the zero-profit wage rule assumption is of course stringent. Hence it is important to consider alternative mechanisms of determining initial wage offers, and to assess whether the modeling results of the paper are robust to such alternative specifications.

The model implicitly assumes that firms have all the bargaining power to set wages once an employment relationship commences. This assumption about firm bargaining power is counterbalanced by the zero-profit wage rule. The latter assumption of course removes all monopsony power and shifts expected lifetime rents to the worker whenever the worker accepts a new zero-profit wage offer. So the wage setting mechanism displays extreme elements of both firm bargaining power and competition for prospective workers. However, if the model dispenses with the competitive assumption without limiting this bargaining power then firms will have both the bargaining power to set wages and monopsony power to determine initial wages, like for example, in Postel-Vinay and Robin (2002). A reasonable alternative is to relax the competitive assumption without allowing the firm complete monopsony power. One specific idea is to assume that outside firms have incomplete information about the match quality of a worker in the incumbent firm. Then the assumption that firms have monopsony power to set initial wage offers is counterbalanced by the fact that outside firms are disadvantaged on account of this information asymmetry. The existence of equilibrium and implications for wage and turnover dynamics of this model setup are currently under investigation (Gerratana and Munasinghe 2005).

In addition to these non-cooperative solutions, there is of course an extensive history of cooperative solution concepts in the literature on rent sharing. The important point is

[13] Such provision of training of course should be seen as part of the contractual relationship between the worker and the firm, as is the wage, which in turn would impose constraints on the patterns of correlations between initial wages and growth rates. See Section 4.4 for further discussion of this issue.

that irrespective of the specific solution we adopt for how initial wage offers are determined, including the above proposal, the salient feature of the model that generates wage and turnover dynamics is the assumed Bertrand bidding competition between the incumbent firm and outside firm. Hence, as long as within-job wage dynamics are determined by a policy of wage renegotiation the key qualitative implications related to within-job wage and turnover dynamics, including serial correlation of wages, are robust to alternative determinations of initial wage offers.

Different solution concepts for wage offers, however, will clearly impact the size of the up-front wage premium, and also, in many cases, the efficiency of labor markets. In particular, if firms offer non-competitive wages that imply positive profits then the initial wage premium will decline and the expected size of within-job wage growth will be correspondingly larger. Hence one of the modeling results of the paper that will change with the introduction of non-competitive wage offers is the prediction of weak tenure effects on wages (see Section 4.1 below). But note that empirical estimates of wage returns on tenure are modest, and thus the zero-profit wage rule is certainly more compatible with this finding than alternative wage rules that would allow for a more substantial rate of wage growth on the job.

3.4.3 Exogenous Search

The formal incorporation of search effort into the current framework adds considerable complexity to the modeling details, and hence this important extension is left to a future research project. However, it is important to highlight some likely ramifications of endogenous search even though these results are not formally derived here.

Since the model assumes a *constant* arrival rate of outside job offers, search effort is not endogenously determined in the model. But of course workers are likely to influence the arrival rate of outside job offers by searching more or less intensely, and their optimal effort level will be determined by a benefit-cost analysis of search effort. Under standard assumptions – an increasing marginal cost function – search effort will be a function of the wage level since the current wage is a sufficient statistic for job value. Since lower wages imply higher marginal gains to search, optimal search effort will be a negative function of current wages. If search effort is a direct function of wages then endogeneity of search will not alter the qualitative results of the paper because the wage level would still remain a sufficient statistic for job value. As a consequence the model implications that hold current wages constant are likely to remain robust with endogenous search effort.

Although endogenous search is unlikely to change the qualitative results of the paper, it will affect some of the welfare properties of the wage renegotiation policy. In particular, as Mortensen (1978) observed, a wage policy of matching outside offers will lead to inefficiently high levels of search intensity even though the turnover rule remains optimal. Since intensity of a worker's search effort is a function only of the wage the worker receives, optimal search effort will be inefficiently high because the worker does not take into consideration the capital loss incurred by the firm. In addition, if the offer arrival rate is a function of the wage level, then wages (turnover) would increase (decrease) more rapidly at lower wage levels and less rapidly at high wage levels than would be predicted by a constant offer arrival rate.

A final observation is that with endogenous search if workers sample multiple firms then dispensing with the zero-profit wage rule will not lead to full monopsony power since competition will imply that the first best firm offer the zero-profit wage of the second best firm.

4 Model Implications

This section derives various model implications that are consistent with the empirical findings detailed in Section 2. The first set of implications derived in Section 4.1 relates to the evolution of wages and turnover with tenure. Section 4.2 explicitly incorporates heterogeneity of productivity growth rates and derives the key intermediate result that the highest outside wage offer a firm matches is higher in high growth jobs than in low growth jobs, holding match value constant. Section 4.3 compares within-job wage growth and turnover rates across high and low growth jobs. Section 4.4 shows that the covariance of successive wage increases is negative for a given productivity profile, whereas the unconditional covariance is indeterminate. Section 4.5 concludes with a discussion of some shortcomings of the paper.

4.1 Wage and Turnover Dynamics

Two immediate implications of the model are: mean wages increase and turnover rates decrease with tenure. These results follow directly from the fact that the highest outside wage offer a firm is willing to match is increasing in p and g. These results are formally stated in Proposition 1. Denote $\widehat{w}_t$ as the highest outside wage offer the firm would match at time t where $\widehat{w}_t \equiv \mathbf{w}(g^t\sigma)$, and $\overline{w}_t$ as the mean or expected wage at time t.

PROPOSITION 2. (1) Mean wages increase with job tenure: $\overline{w}_{t+1} > \overline{w}_t, \forall t > 0$.
(2) Turnover rates decrease with job tenure: $1 - F(\widehat{w}_{t+1}) < 1 - F(\widehat{w}_t), \forall t \geq 0$.

PROOF. See Appendix.

Recall that the zero-profit equilibrium wage function $\mathbf{w}(\cdot)$ – i.e. the wage function that determines the initial wage and the highest outside wage offers the firm is willing to match in subsequent time periods – is increasing in p for any given g. Hence this highest outside wage offer the firm is willing to match increases with tenure because productivity increases on the job. As a consequence the mean wage increases and turnover decreases with tenure. These results are graphically illustrated in Figure 1. Note however that the mean wage increase is smaller than the corresponding increase in the highest outside wage offer the firm is willing to match since the latter is only the upper bound for a renegotiated wage contract. But the decrease in the turnover rate corresponds directly to the corresponding increase in the highest outside wage offer the firm is willing to match. The model therefore implies the standard dual tenure effects, but the positive tenure effect on wages, unlike the negative tenure effect on turnover, is attenuated due to the wage renegotiation policy. These asymmetric tenure effects are consistent with the findings of quantitatively small (and not always significant) positive tenure effects on wages and large (and always significant) negative tenure effects on turnover. Hence a small estimated wage return on tenure should not be interpreted as necessarily implying a diminished role for specific skill accumulation on the job. The negative tenure effect on turnover is in fact the more appropriate gauge of specific skill accumulation.[14]

The model also implies that tenure will be negatively correlated with turnover even if the current wage is held constant. Recall, turnover is determined by the highest outside wage offer the firm is willing to match but the current wage always lies somewhere between this "highest" wage and the initial zero-profit equilibrium wage. For example, if wages remain unchanged (due to a low outside wage offer), the turnover rate still falls with tenure because the firm will match a higher outside wage offer in the next period. Hence turnover will decrease with tenure even if wages are held constant, an implication consistent with the finding that turnover rates decrease with tenure holding wages constant (Topel and Ward 1992). As mentioned in Section 2, matching models are unable to reconcile this fact because they predict turnover will increase with tenure if wages are held constant.

4.2 High and Low Growth Jobs

This section introduces high and low productivity growth jobs and derives an intermediate result that underpins various corollary results on wage growth, turnover, and serial correlation of wage increases. First, denote high and low growth jobs as $\sigma_0^H \equiv (p^H, g^H)$ and $\sigma_0^L \equiv (p^L, g^L)$, respectively, and note that $g^H > g^L$. Assume that both jobs have the same match value at the time of job start, i.e. $W(\sigma_0^H) = W(\sigma_0^L)$. Since $W(\sigma_0^H) = V(\mathbf{w}(\sigma_0^H))$ and

[14] Empirical estimates of wage returns on tenure hold experience constant since within-job wage increases also include returns on general work experience. Since the model presented here does not allow for general productivity increases, it does not generate wage returns to experience, and hence within-job wage increases are only due to returns on tenure.

$W(\sigma_0^L) = V(\mathbf{w}(\sigma_0^L))$, and V is a monotonically increasing function of wages, both jobs have the same equilibrium wage, i.e. $\mathbf{w}(\sigma_0^H) = \mathbf{w}(\sigma_0^L)$. This implies that in all subsequent time periods the highest outside wage offer a firm is willing to match is higher in the high growth job than in the low growth job. This result is formally stated below. Let $\sigma_t^J \equiv ((g^J)^t p^J, g^J)$ for $J = H$ and L, and for notational simplicity denote $\widehat{w}_0^H (\equiv \mathbf{w}(\sigma_0^H))$ and $\widehat{w}_0^L (\equiv \mathbf{w}(\sigma_0^L))$ as the equilibrium wages, and $\widehat{w}_t^H (\equiv \mathbf{w}(\sigma_t^H))$ and $\widehat{w}_t^L (\equiv \mathbf{w}(\sigma_t^L))$ as the highest outside wage offers a firm is willing to match at time t, in the high and low growth jobs, respectively.

LEMMA 5. If $\widehat{w}_0^H = \widehat{w}_0^L$ then $\widehat{w}_t^H > \widehat{w}_t^L, \forall t > 0$.

PROOF. Since match value is a function of the growth rate it follows that:

$$\text{If } W(\sigma_0^H) = W(\sigma_0^L) \text{ then } W(\sigma_t^H) > W(\sigma_t^L), \forall t > 0,$$

since $g^H > g^L$. The match value of the high growth job increases faster than the match value of the low growth job. Note $\widehat{w}_t^H$ and $\widehat{w}_t^L$ must satisfy $W(\sigma_t^H) = V(\widehat{w}_t^H)$ and $W(\sigma_t^L) = V(\widehat{w}_t^L)$, respectively. Since $W(\sigma_t^H) > W(\sigma_t^L)$ and V is monotonically increasing in wages, $\widehat{w}_t^H > \widehat{w}_t^L$, $\forall t > 0$. ■

This result underpins the various model implications related to wage growth, turnover, and serial correlation of wage increases derived in the next two subsections.

4.3 Wage Growth and Turnover

The following proposition states that the mean wage is higher and turnover is lower in the high growth job than in low growth job, holding the initial zero-profit equilibrium wage constant – i.e. $\widehat{w}_0^H = \widehat{w}_0^L$. Denote $\overline{w}_t^H$ and $\overline{w}_t^L$ as the mean wages at time t in the high and low growth job, respectively.

PROPOSITION 3. (1) Mean wage is higher in the high growth job than in the low growth job: $\overline{w}_t^H > \overline{w}_t^L, \forall t > 0$. (2) Turnover rate is lower in the high growth job than in the low growth job: $1 - F(\widehat{w}_t^H) < 1 - F(\widehat{w}_t^L), \forall t > 0$.

PROOF. See Appendix.

Both items in Proposition 3 follow from Lemma 5. The turnover result needs the additional assumption that workers in both jobs sample outside wage offers from the same distribution in every period. These results are graphically illustrated in Figure 2. Note, although both jobs have the same match value (and hence the same initial zero-profit equilibrium wages at the time of job start), it does not imply that the expected sum of productivities in the two jobs are the same. Recall that match value refers to the present value of lifetime productivity that includes not only the incumbent job but also future jobs. In the high growth job expected productivity in the current firm is larger than it is in the low growth job, holding match value constant.

The corollary of Proposition 3 – namely, that wage growth on a job is negatively related to turnover – addresses a key finding in the literature. The precise finding is that the initial wage and current wage jointly have significant positive and negative effects on turnover, respectively (Topel and Ward, 1992). Although the current wage is not a precise proxy for current match value, if the initial "equilibrium" wage is held constant then the current wage is a proxy for wage growth. Similarly, if the current wage is held constant then the initial wage is also a proxy, albeit negatively, for wage growth. Since the model predicts that wage growth reduces turnover, the observed negative effect of initial wages on turnover and positive effect of current wages on turnover are entirely consistent with the model.

4.4 Serial Correlation of Wage Increases

Although productivity increases on the job are deterministic wage increases on the job follow a stochastic process, and hence serial correlation of wage increases are more complex than the assumed serial correlation of productivity increases. In particular, for a given productivity profile the covariance of successive wage increases is negative, whereas the same covariance without the conditioning is indeterminate. The latter result resolves the paradox on the lack of evidence of heterogeneity of permanent rates of wage growth among jobs.

For expositional convenience, consider the wage outcomes in two consecutive time periods. Let $\widehat{w}_1$ be a random variable and define a second random variable as $\widehat{w}_2 = (1+\alpha)\widehat{w}_1$, where $\alpha > 0$. Interpret $\widehat{w}_1$ and $\widehat{w}_2$ as the highest outside wage offers that a firm matches in the two periods immediately following employment. Denote $\widehat{w}_0$ as the initial equilibrium wage, and note $\widehat{w}_0 < \widehat{w}_1 < \widehat{w}_2$. If $\widehat{w}_0$ is a constant then sequences given by $\{\widehat{w}_0, \widehat{w}_1, \widehat{w}_2\}$ mimics various productivity profiles of jobs with equivalent match values. A higher draw from $\widehat{w}_1$ simply refers to a steeper productivity profile – i.e., to a higher g. The covariance of successive increases in the highest outside wage offers a firm is willing to match is positive by construction: $Cov(\widehat{w}_1 - \widehat{w}_0, \widehat{w}_2 - \widehat{w}_1) = \alpha Var(\widehat{w}_1) > 0$. Note the reason for this positive covariance is the assumption of a positive covariance of successive increases in productivity.

Next, let X_1 and X_2 be the wage offers in periods 1 and 2, respectively, from a stationary distribution. Let w_1 and w_2 be the observed wages in periods 1 and 2 due to wage renegotiation. In the first period, if $X_1 \leq \widehat{w}_0$ then $w_1 = \widehat{w}_0$ and if $\widehat{w}_0 < X_1 \leq \widehat{w}_1$ then $w_1 = X_1$; in the second period, if $X_2 \leq w_1$ then $w_2 = w_1$ and if $w_1 < X_2 \leq (1+\alpha)\widehat{w}_1$ then $w_2 = X_2$. If either $X_1 > \widehat{w}_1$ or $X_2 > \widehat{w}_2$ then there are no within-job wage sequences because the worker would have quit and gone to a new firm. The following proposition states the main result.

PROPOSITION 4. (1) For a given productivity profile the covariance of successive wage increases is negative: $Cov(w_1 - \widehat{w}_0, w_2 - w_1) < 0$, for a given $\{\widehat{w}_0, \widehat{w}_1, \widehat{w}_2\}$. (2) The unconditional covariance however is indeterminate: $Cov(w_1 - \widehat{w}_0, w_2 - w_1) \gtreqless 0$.

PROOF. See Appendix.

The intuition behind the first item of this proposition is that the first period wage is the lower bound for the second period wage. So if the wage increase is small in the first period then the scope for wage increase in the second period is relatively high, and vice versa. So, expected wage increase in the second period is negatively related to the first period wage increase, implying that within-job wage increases are negatively correlated for any given productivity profile (see Figure 3).

The second item of Proposition 4 may appear counter intuitive since for any given productivity profile the covariance is negative. However, if observations from high and low growth jobs are combined the covariance between first period wage increases and second period wage increases becomes indeterminate. Since covariance measures the linear association between the deviations of two random variables from their respective means, the mean wage increases in the first and second period change when populations with different growth rates are combined. As a result the covariance of successive wage increases becomes indeterminate. Figure 4 attempts to illustrate.

The exact sign of this unconditional covariance for a given ϕ will depend on which of the following two countervailing forces dominate. The first is of course the negative correlation of successive wage increases for a given productivity profile – i.e. item (1) of Proposition 4. A lower growth rate is likely to result in a smaller negative correlation because it makes zero wage increases in successive periods more likely. Also a higher match value will tend to reduce this negative correlation because a high match value implies a lower probability of receiving a higher outside wage offer. These factors that make small or zero wage increases in successive periods more likely will tend to attenuate this negative correlation. Put differently, a productivity profile with a high growth rate or low match value will imply a stronger negative correlation of successive wage increases.

The second, positive effect is due to the extent of heterogeneity of productivity growth rates holding match value constant. Clearly the means of wage increases in successive periods will be positively correlated given heterogeneity of productivity growth rates. In terms of the basic elements of the model, if p and g are positively correlated then holding match value constant would imply little variation in observed growth rates, which in turn would attenuate this positive effect.[15] However, if p and g are negatively correlated then even holding match value constant would generate a large variation in g, and thus a large positive effect on the unconditional covariance term. Clearly the joint distribution of p and g will determine the size of this positive effect on the unconditional covariance. Hence the exact sign of the unconditional covariance of wage increases in successive periods will depend on which of these two countervailing forces dominate.

To address the empirical results on serial correlation of wage increases, note one implication of item (1) in Proposition 4 is that if heterogeneity of permanent rates of wage growth among jobs is unimportant, as Topel and Ward (1992) claim, then the model predicts a negative covariance of successive wage increases, which they do not find in their data. The key theoretical point, however, is the indeterminacy of serial correlation of wage increases despite the fact that productivity increases are serially correlated by construction. Therefore, serial correlation of wage increases is an inconclusive test of differences in permanent rates of wage growth among jobs. Hence studies that fail to find positive serial correlation of wage increases such as Topel (1991) and Topel and Ward (1992) do not necessarily present evidence against the hypothesis of wage growth heterogeneity. More importantly, Propositions 3 and 4 jointly solve one of the empirical riddles that motivated the paper: past wage growth reduces turnover and yet the same data show no evidence of serial correlation of wage increases.

4.5 Some Shortcomings

Since one of the main motivations for this study is to replicate properties of individual wage and turnover data, the absence of a quantitative evaluation of the model is a limitation of the

[15] The correlation between p and g, or its equivalent counterparts, has been investigated both theoretically and empirically. As mentioned earlier, if g was endogenously determined by p like in Jovanovic (1979b) then holding match value constant would imply no variation in g at all. On the other hand, different on-the-job learning opportunities combined with equilibrium considerations could imply a negative correlation between between p and g. For example, Becker (1975) and Mincer (1993) have argued that labor mobility will lead to equalization of present values among jobs with different learning opportunities, and Hause (1980), using Swedish data, estimated a wage model that implied a negative correlation between the individual constant term and slope. These latter studies suggest a negative correlation between p and g.

A similar idea can also be found in the literature on the adoption of new technologies. As Parente (1993) writes, "The firm faces a trade-off in its choice of technologies to adopt. The more advanced the new technology is, the greater its productive potential, but smaller the firm's starting level of expertise in that technology." The comparative static exercise in this paper also implies a similar trade-off between current productivity and growth rate of productivity.

paper. In particular, the quantitative effects of tenure on wages and turnover, and the sign of the covariance of wage increases in adjacent time periods clearly depend on the distribution of p and g among jobs. Hence a more quantitative analysis of the joint distributions of p and g that could generate empirically sensible results is an important extension and validation of the model elaborated in this paper.

The model presented here also does not address other well documented facts about wages and turnover. For example, although the evidence on mobility wage gains conditional on a quit is largely positive, a substantial fraction of job-to-job changes are associated with wage cuts, which is inconsistent with the model. A model where a firm pays a worker her marginal product in every period could generate such a result because a worker moving to a high growth job would be willing to take a wage cut in anticipation of receiving higher wages in the future. Moreover, the assumed downward wage rigidity of the model is inconsistent with evidence that shows within-job wage cuts are not uncommon (Baker, Gibbs and Holmstrom 1994) and negative wage returns on tenure (Ransom 1993).

The model is also silent about wage returns on general labor market experience, and hence the paper does not address the voluminous empirical literature on estimating wage returns on general human capital. Furthermore, the theory only models job-to-job transitions and does not address unemployment, which is a central topic in the search literature.

5 Related Theory

The model in this paper is closely related to the theoretical literature on search-and-matching, specific human capital, and wage renegotiation. Since Becker's original idea of sharing costs and returns of firm specific investments as a means of providing mutual insurance to each party's investment, the problem of wage determination has been well known (Becker 1962; Parsons 1972; Hashimoto 1981). Although the inherent inefficiency of Becker-type sharing rules was recognized from the beginning, Mortensen (1978) is the first to explicitly consider employment agreements that would induce both workers and employers to pursue efficient or joint wealth maximizing search strategies. In particular, Mortensen considers two such wage setting mechanisms: first, matching alternative offers obtained by one's partner, and second, *ex ante* agreement by each party to compensate the other as a precondition for separation. He shows counteroffer matching implies an efficient turnover rule if on-the-job search is exogenous. With endogenous search effort, although the turnover rule remains efficient, each party still has an incentive to search too intensively. The second employment agreement of contingent compensation leads to both an efficient turnover rule and efficient search effort, and thus it is joint wealth maximizing.

The wage setting mechanism in this paper is based on Mortensen's idea of matching alternative offers.[16] In addition, by considering only the case of exogenous worker search, the form of the employment agreement adopted here is a simpler version of Mortensen. As a consequence, the analysis of wage dynamics is more tractable, and the turnover rule implied by wage renegotiation is efficient for exactly the reasons expounded by Mortensen. The key difference with Mortensen's seminal work is the analysis of the equilibrium wage function. Although in principle Mortensen's framework might be general enough to allow match productivity to vary over time, this possibility is not developed into an analysis of wage determination. In particular, the absence of an equilibrium wage function implies that Mortensen's framework cannot generate wage dynamics. In contrast, the model here by imposing structure on productivity changes on the job and by introducing a competitive setting is able to explicitly derive an equilibrium wage function. Of course, Mortensen's objective – to analyze efficient employment agreements in the presence of specific rents and search frictions – is different from the more immediate objectives of this paper – to develop a theory consistent with a wide range of facts related to wage and turnover dynamics.

In a recent work, Postel-Vinay and Robin (2002) also adopt wage renegotiation as their wage setting mechanism. As a consequence, their model and the one here share some common features including within-job wage dynamics and efficient turnover. However some implications differ across these models. The key difference, apart from the fact that their model addresses unemployment, is the determination of first period equilibrium wages. In Postel-Vinay and Robin, firms pay unemployed workers only their reservation wage, and if workers are employed then firms pay only the minimum wage just sufficient to lure workers away from their incumbent employers. So firms collect maximal rents in contrast to the model here where all expected rents go to the worker. As Postel-Vinay and Robin demonstrate there will be wage growth on the job even though productivity is flat because workers are not paid their marginal product but employers will match outside wage offers when required. Therefore wage growth can exceed productivity growth in contrast to what the model here predicts. Moreover, their model allows for wage cuts when workers voluntarily move to a firm that is likely to be more aggressive in matching outside offers in the future. However, heterogeneity of firm-specific productivity growth is not a feature of Postel-Vinay and Robin's model, and thus it is not explicitly designed to address implications related to wage growth and turnover, and serial correlation of wage increases.

[16] Since Mortensen, the idea of wage renegotiation has been widespread. For example, the wage setting mechanism of Harris and Holmstrom (1982) is based on renegotiation: firms promise to pay a rigid wage until the worker receives a better offer from the market at which point the old contract is cancelled and a new contract is made which matches the market offer. Also Malcomsom's (1997) review article presents various other applications of wage renegotiation.

The introduction of productivity growth into the Mortensen type wage renegotiation framework makes this paper similar to Jovanovic (1979b) since it is one of the first theoretical articles explicitly to integrate specific human capital theory into a search-and-matching framework. Put differently, this paper can be viewed as a "Jovanovic (1979b) meets Mortensen (1978)" type model. In Jovanovic's model, the level of match quality determines expected job duration which in turn jointly determines optimal search effort and investment in firm specific human capital. Thus match quality is positively correlated with productivity growth on the job. Jovanovic's central result is that turnover declines with tenure. The key theoretical difference, however, is in the wage setting mechanism. In Jovanovic's model the employer makes a wage offer to the worker that is equal to marginal product. The justification for such a wage policy is based on reputation repercussions. As Jovanovic says, "employers offering wages below marginal product will acquire bad reputations and will consequently not be sampled by workers" (p. 1249, Jovanovic 1979b). But firms unable to commit to future wage increases will have to offer time consistent wage policies. The policy of wage renegotiation considered here is immune to charges of time inconsistency.

The on-the-job search aspect of the model is based on imperfect information about the location of the best match as in Burdett (1978). In all adaptations of search-and-matching models, including Burdett's, productivity level is a sufficient statistic for job value. In this paper, however, each worker-firm pair is characterized by an idiosyncratic firm-specific productivity profile, and the match value of an employment relationship increases with time on the job due to productivity growth. Embedding firm-specific productivity growth within a job matching framework generates implications for within-job wage and turnover dynamics, unlike Burdett's model that only generates implications for wage changes due to job switches and turnover dynamics over the life cycle.

This paper falls within the literature on search-theoretic equilibrium models of labor markets, and it is closely allied with various recent models of on-the-job search such as Burdett and Mortensen (1998), Postel-Vinay and Robin (2002), and Burdett and Coles (2003), to mention only a few. All these models, like the model in this paper, are based on non-cooperative solutions to the problem of wage determination when employment relationships generate firm-specific rents. These models also generate wage dispersion even when workers and firms are homogeneous, and hence they contrast sharply with other standard models of search that is based on the Nash bargaining solution to the surplus-splitting problem.[17] Note, however, that in a recent paper Shimer (2004) shows that Nash bargaining can lead to similar wage dispersion results as the wage posting model of Burdett and Mortensen (1998).

Some of the best known theories of compensation and labor mobility incorporate vari-

[17] See Rogerson et al. (2004) for a detailed survey of this literature on search-theoretic models.

ous features of “learning” over the duration of an employment relationship. For example, Jovanovic’s (1979a) mismatch theory of turnover is based on learning about match quality. Other seminal contributions to the theory of wage dynamics – e.g. Harris and Holmstrom (1982), Waldman (1984), Ricart I Costa (1988), and Farber and Gibbons (1996) – are based on learning about worker ability. The model here does not include any such feature of learning. However, it might be worth considering whether learning about productivity profiles (i.e. treating a match as an experience good) or learning about unknown worker ability might lead to other potentially interesting implications.

On a related note, two recent articles by Gibbons and Waldman (1999) and Chiappori et al. (1999) have proposed models of learning about worker ability to explain a host of findings on wage and promotion dynamics. The Gibbons-Waldman model is specifically designed to explain various aspects of internal labor markets documented by Baker et al. (1994). As mentioned earlier in Section 2, Chiappori et al. derive a late-beginner property of models characterized by learning and downward wage rigidity, and they confirm various predictions related to wage and promotion dynamics using personnel data on executives of a French state-owned firm.[18] Unlike the model presented here, neither of these models focus on firm level turnover. However, an open question is whether promotion dynamics can be derived in the model presented here by incorporating job levels. If firm specific skills are required to move from one job level to the next higher job level then wage increases are likely to predict future promotions, a result explicitly derived in Gibbons and Waldman. But, if promotions also signal higher general skills, such as managerial talent to competitor firms, then turnover implications following a promotion are likely to be amended. More work on the empirical links between promotions, wages and turnover will suggest whether the framework developed in this paper might be suitable to analyze internal labor market phenomena.

6 Conclusion

The paper presents a theory of compensation based on search-and-matching, firm-specific human capital, and wage renegotiation. In summary, wage increases occur because firms match outside wage offers, and on-the-job search provides the impetus for within-job wage increases. Average wage increases are higher in high growth jobs because they generate more firm specific rents. Past wage growth on a job negatively predicts quits because turnover is

[18] The model here also generates a result very similar to the late-beginner property: w_1 is negatively related to w_3 if w_2 is held constant. If w_1 is lower because of a low match value and w_2 is relatively high because of a high outside wage offer then the predicted wage increase in the following time period is lower (see first item of Proposition 3). This implies a lower w_3. The only difference is that LDR models also hold w_0 constant, whereas the model here presumes that w_1 is the zero-profit equilibrium wage.

lower in high growth jobs than in low growth jobs. A further implication of the model is that for a given productivity profile the covariance of successive wage increases is negative, whereas the same covariance without the conditioning is indeterminate. These results explain a wide range of findings on wage and turnover dynamics.

7 Appendix

PROOF OF LEMMA 1

Define the operator $\Gamma W(\sigma)$ from space $C(\Sigma)$ of continuos functions with domain Σ as: $\Gamma W(\sigma) = p + \beta \left\{ W(g\sigma)\phi(\widetilde{\sigma}|W(\widetilde{\sigma}) \leq W(g\sigma)) + \int_{\{\widetilde{\sigma}|W(\widetilde{\sigma})>W(g\sigma)\}} W(\widetilde{\sigma})d\phi(\widetilde{\sigma}) \right\}$. The space of $C(\Sigma)$ is a Banach space (see for example, Stokey and Lucas, 1989, Chapter 3). $\Gamma : C(\Sigma) \to C(\Sigma)$. Moreover, since $\Gamma W(\sigma)$ is a contraction, the contraction mapping theorem implies the existence and uniqueness of $W(\sigma)$ given by:

$$W(\sigma) = p + \beta \left\{ W(g\sigma)\phi(\widetilde{\sigma}|W(\widetilde{\sigma}) \leq W(g\sigma)) + \int\limits_{\{\widetilde{\sigma}|W(\widetilde{\sigma})>W(g\sigma)\}} W(\widetilde{\sigma})d\phi(\widetilde{\sigma}) \right\}.$$

The stationary policy correspondence can be represented as a preference relation over Σ (i.e. a complete order): $\sigma \succ \sigma'$ iff $W(\sigma) > W(\sigma')$. Simple inspection shows that $W(\sigma)$ is strictly increasing in p and g.[19]

PROOF OF LEMMA 2

To show that $\mathbf{w}(\sigma)$ is a monotone transformation of $W(\sigma)$ consider $\sigma, \sigma' \in \Sigma$. Then we need to show: (1) If $W(\sigma) = W(\sigma')$ then $\mathbf{w}(\sigma) = \mathbf{w}(\sigma')$, and (2) if $W(\sigma) > W(\sigma')$ then $\mathbf{w}(\sigma) > \mathbf{w}(\sigma')$.

(1) For $\sigma, \sigma' \in \Sigma$, if $W(\sigma) = W(\sigma')$ then $\mathbf{w}(\sigma) = \mathbf{w}(\sigma')$.

First note that:

$$W(\sigma) = p + \beta \left\{ W(g\sigma)\phi(\widetilde{\sigma}|W(\widetilde{\sigma}) \leq W(g\sigma)) + \int_{\{\widetilde{\sigma}|W(\widetilde{\sigma})>W(g\sigma)\}} W(\widetilde{\sigma})d\phi(\widetilde{\sigma}) \right\}, \text{ and}$$

$$W(\sigma') = p + \beta \left\{ W(g'\sigma')\phi(\widetilde{\sigma}|W(\widetilde{\sigma}) \leq W(g'\sigma')) + \int_{\{\widetilde{\sigma}|W(\widetilde{\sigma})>W(g'\sigma')\}} W(\widetilde{\sigma})d\phi(\widetilde{\sigma}) \right\}.$$

Hence $W(\sigma) - W(\sigma') = (p - p') + \beta X$, where

$$X = W(g\sigma)\phi(\widetilde{\sigma}|W(\widetilde{\sigma}) \leq W(g\sigma)) + \int_{\{\widetilde{\sigma}|W(\widetilde{\sigma})>W(g\sigma)\}} W(\widetilde{\sigma})d\phi(\widetilde{\sigma}) - W(g'\sigma')\phi(\widetilde{\sigma}|W(\widetilde{\sigma}) \leq W(g'\sigma')) - \int_{\{\widetilde{\sigma}|W(\widetilde{\sigma})>W(g'\sigma')\}} W(\widetilde{\sigma})d\phi(\widetilde{\sigma}).$$

[19] Only a proof sketch is provided here since it follows standard dynamics programming techniques.

Also note since

$$\mathbf{w}(\sigma) = p + \beta \left\{ \begin{array}{l} (W(g\sigma) - W(\sigma))\phi(\widetilde{\sigma}|W(\widetilde{\sigma}) \leq W(\sigma)) \\ + \int_{\{\widetilde{\sigma}|W(\sigma)<W(\widetilde{\sigma})\leq W(g\sigma)\}} (W(g\sigma) - W(\widetilde{\sigma}))d\phi(\widetilde{\sigma}) \end{array} \right\}$$

that $\mathbf{w}(\sigma) - \mathbf{w}(\sigma') = (p - p') + \beta Z$, where

$$\begin{aligned} Z = {} & W(g\sigma)\phi(\widetilde{\sigma}|W(\widetilde{\sigma}) \leq W(g\sigma)) - W(\sigma)\phi(\widetilde{\sigma}|W(\widetilde{\sigma}) \leq W(\sigma)) \\ & - \int_{\{\widetilde{\sigma}|W(\sigma)<W(\widetilde{\sigma})\leq W(g\sigma)\}} W(\widetilde{\sigma})d\phi(\widetilde{\sigma}) - W(g'\sigma')\phi(\widetilde{\sigma}|W(\widetilde{\sigma}) \leq W(g'\sigma')) \\ & + W(\sigma')\phi(\widetilde{\sigma}|W(\widetilde{\sigma}) \leq W(\sigma')) + \int_{\{\widetilde{\sigma}|W(\sigma')<W(\widetilde{\sigma})\leq W(g'\sigma')\}} W(\widetilde{\sigma})d\phi(\widetilde{\sigma}) \end{aligned}$$

If $W(\sigma) = W(\sigma')$ then $(p-p') = -\beta X$. Substituting this into the equilibrium wage difference equation gives $\mathbf{w}(\sigma) - \mathbf{w}(\sigma') = \beta(Z - X)$. Hence if $(Z - X) = 0$ then we are done. Note that $Z - X$ is given by:

$$\begin{aligned} Z - X = {} & W(g\sigma)\phi(\widetilde{\sigma}|W(\widetilde{\sigma}) \leq W(g\sigma)) - W(\sigma)\phi(\widetilde{\sigma}|W(\widetilde{\sigma}) \leq W(\sigma)) \\ & - \int_{\{\widetilde{\sigma}|W(\sigma)<W(\widetilde{\sigma})\leq W(g\sigma)\}} W(\widetilde{\sigma})d\phi(\widetilde{\sigma}) - W(g'\sigma')\phi(\widetilde{\sigma}|W(\widetilde{\sigma}) \leq W(g'\sigma')) + W(\sigma')\phi(\widetilde{\sigma}|W(\widetilde{\sigma}) \leq W(\sigma')) \\ & + \int_{\{\widetilde{\sigma}|W(\sigma')<W(\widetilde{\sigma})\leq W(g'\sigma')\}} W(\widetilde{\sigma})d\phi(\widetilde{\sigma}) - W(g\sigma)\phi(\widetilde{\sigma}|W(\widetilde{\sigma}) \leq W(g\sigma)) - \int_{\{\widetilde{\sigma}|W(\widetilde{\sigma})>W(g\sigma)\}} W(\widetilde{\sigma})d\phi(\widetilde{\sigma}) \\ & + W(g'\sigma')\phi(\widetilde{\sigma}|W(\widetilde{\sigma}) \leq W(g'\sigma')) + \int_{\{\widetilde{\sigma}|W(\widetilde{\sigma})>W(g'\sigma')\}} W(\widetilde{\sigma})d\phi(\widetilde{\sigma}). \end{aligned}$$

Simplifying yields

$$\begin{aligned} Z - X = {} & W(\sigma')\phi(\widetilde{\sigma}|W(\widetilde{\sigma}) \leq W(\sigma')) + \int_{\{\widetilde{\sigma}|W(\widetilde{\sigma})>W(\sigma')\}} W(\widetilde{\sigma})d\phi(\widetilde{\sigma}) \\ & - W(\sigma)\phi(\widetilde{\sigma}|W(\widetilde{\sigma}) \leq W(\sigma)) - \int_{\{\widetilde{\sigma}|W(\widetilde{\sigma})>W(\sigma)\}} W(\widetilde{\sigma})d\phi(\widetilde{\sigma}). \end{aligned}$$

Since $W(\sigma') = W(\sigma)$ clearly $Z - X = 0$.■

(2) For $\sigma, \sigma' \in \Sigma$, if $W(\sigma) > W(\sigma')$ then $\mathbf{w}(\sigma) > \mathbf{w}(\sigma')$.

Let $\Delta W = W(\sigma) - W(\sigma')$ and $\Delta \mathbf{w} = \mathbf{w}(\sigma) - \mathbf{w}(\sigma')$. Note that $\Delta W - \Delta \mathbf{w} = \beta(X - Z)$. If $X - Z < W(\sigma) - W(\sigma')$ then $\Delta W - \Delta \mathbf{w} < \Delta W$ and hence $\Delta \mathbf{w} = \mathbf{w}(\sigma) - \mathbf{w}(\sigma') > 0$, which completes the proof.

First note

$$\begin{aligned} X - Z = {} & W(\sigma)\phi(\widetilde{\sigma}|W(\widetilde{\sigma}) \leq W(\sigma)) + \int_{\{\widetilde{\sigma}|W(\widetilde{\sigma})>W(\sigma)\}} W(\widetilde{\sigma})d\phi(\widetilde{\sigma}) \\ & - W(\sigma')\phi(\widetilde{\sigma}|W(\widetilde{\sigma}) \leq W(\sigma')) - \int_{\{\widetilde{\sigma}|W(\widetilde{\sigma})>W(\sigma')\}} W(\widetilde{\sigma})d\phi(\widetilde{\sigma}). \end{aligned}$$

Since $W(\sigma) > W(\sigma')$ we can simplify further and write

$$\begin{aligned} X - Z &= [W(\sigma) - W(\sigma')]\phi(\widetilde{\sigma}|W(\widetilde{\sigma}) \leq W(\sigma')) + \int_{\{\widetilde{\sigma}|W(\sigma')<W(\widetilde{\sigma})<W(\sigma)\}} [W(\sigma) - W(\widetilde{\sigma})]d\phi(\widetilde{\sigma}) \\ &\leq [W(\sigma) - W(\sigma')]\phi(\widetilde{\sigma}|W(\widetilde{\sigma}) \leq W(\sigma)) \leq W(\sigma) - W(\sigma'). \end{aligned}$$

And since $\beta < 1$, $\beta(X - Z) = \Delta W - \Delta\mathbf{w} < \Delta W$. And hence if $\Delta\mathbf{w} > 0$, which implies $\mathbf{w}(\sigma) > \mathbf{w}(\sigma')$.■

PROOF OF PROPOSITION 2

To show that average wages increase with tenure, first define the expected wage at time $t+1$ conditional on wage w_t at time t as:

$$\overline{w}_{t+1}|w_t = \alpha w_t + (1-\alpha)\int_{w_t}^{\widehat{w}_{t+1}} w dF(w) > w_t,$$

where $\alpha = F(w_t)/F(\widehat{w}_{t+1})$. Since $\overline{w}_{t+1}|w_t$ is greater for every possible previous period wage w_t, it follows that $\overline{w}_{t+1} > \overline{w}_t$. The turnover result follows because the highest wage the firm is willing to match increases with tenure, i.e. $\widehat{w}_{t+1} > \widehat{w}_t, \forall t \geq 0$, and F is strictly increasing.

PROOF OF PROPOSITION 3

If the distribution of wages in the high growth job stochastically dominates the wage distribution in the low growth job then the first item of Proposition 3 follows trivially. First define a r.v. X from the distribution as F. Next define two further r.v.s, R^L and R^H, such that:

$$\begin{aligned} R^L &= X \text{ if } a_1 < X \leq b_1 \\ &= a_1 \text{ if } X \leq a_1 \\ &= \infty \text{ if } X > b_1,\, a_1 < b_1 \end{aligned}$$

and similarly for R^H:

$$\begin{aligned} R^H &= X \text{ if } a_2 < X \leq b_2 \\ &= a_2 \text{ if } X \leq a_2 \\ &= \infty \text{ if } X > b_2,\, a_2 < b_2 \end{aligned}$$

Note further that $a_1 < a_2 < b_1 < b_2$. The following lemma claims that R^H stochastically dominates R^L, conditional on both been finite:

LEMMA. $P(W^L > x|W^L < \infty) \leq P(W^H > x|W^H < \infty), \forall x \geq 0.$

Proof. Define

$$
\begin{aligned}
P(R^L > x | R^L < \infty) &= \frac{P(R^L > x, R^L < \infty)}{P(R^L < \infty)} \\
&= \frac{P(R^L > x, X \leq b_1)}{F(b_1)} \\
&= \frac{P(R^L > x, X \leq a_1) + P(R^L > x, a_1 < X \leq b_1)}{F(b_1)} \\
&= \frac{P(a_1 > x, X \leq a_1) + P(X > x, a_1 < X \leq b_1)}{F(b_1)} (\equiv G^L(x))
\end{aligned}
$$

Similarly define the distribution of wages in the high growth job, $P(R^H > x | R^H < \infty)$, as:

$$
\frac{P(a_2 > x, X \leq a_2) + P(X > x, a_2 < X \leq b_2)}{F(b_2)} (\equiv G^H(x))
$$

We now show that for all $x \geq 0$ the inequality in the above Lemma holds. If $x \leq a_1$ then $G^L(x) = G^H(x) = 1$. If $a_1 < x \leq a_2$ then $G^H(x) = 1$. If $a_2 \leq x < b_1$ then

$$
G^L(x) = \frac{F(b_1) - F(x)}{F(b_1)} < \frac{F(b_2) - F(x)}{F(b_2)} = G^H(x),
$$

since $b_2 > b_1$. Finally if $b_1 \leq x < b_2$ then $G^L(x) = 0$. Hence the inequality holds for $x \geq 0$, which completes the proof of the lemma. ■

The remainder of the proof proceeds by considering wages in the first time period, and using the above lemma to show that the wage distribution in the high growth job stochastically dominates the wages in the low growth job. Then the lemma is used repeatedly to show that it holds for all time periods.

Consider wages in the high and low growth jobs in the first period: W_1^L, and W_1^H, respectively. In period 0 the wages are the same in both the high and low growth jobs. Hence let $\widehat{w}_0^H = \widehat{w}_0^L = a$, and let the upper barriers: $b_1 = \widehat{w}_1^L < \ b_2 = \widehat{w}_1^H$. W_1^L is thus distributed as $(R^L | R^L < \infty)$ and W_1^H as $(R^H | R^H < \infty)$. Thus from the lemma, W_1^L is stochastically dominated by W_1^H. We can then take copies such that $W_1^L = a_1 < a_2 = W_1^H$. Redefine $b_1 = \widehat{w}_1^L < b_2 = \widehat{w}_1^H$ and proceed again with the lemma to show that W_2^L stochastically dominates W_2^H. By continuing in this manner, we conclude the result holds for all t.

The turnover result follows directly from Lemma 5: $\widehat{w}_t^H > \widehat{w}_t^L, \forall t > 0$, and the fact that workers from both jobs sample outside wage offers from the same distribution.

PROOF OF PROPOSITION 4

We begin with a more formal statement of the correlation between wage increases in adjacent time periods. Let $f(x)$ denote a probability density function on $(0, \infty)$; $f(x) \geq$

$0,\ x \in (0,\infty)$ and $\int_0^\infty f(x)dx = 1$. Let $F(x) = \int_0^x f(y)dy$, and let $\bar{F}(x) = 1 - F(x)$,where F is the underlying wage offer distribution.

Let $\{X_n : n \geq 1\}$ denote an independent and identically distributed sequence of r.v.s. distributed as F: $P(X \leq x) = F(x),\ x \geq 0$. Let $0 < p_0 < p_1 < \cdots$ denote an increasing sequence of numbers tending to ∞. Let $V_0 = p_0,\ V_1 = (X_1 \mid X_1 < p_1)$ and in general $V_n = (X_n \mid X_n < p_n),\ n \geq 1$,which means that V_n is an independent copy of a wage X conditional on it falling in the interval $(0, p_n)$.

Now define $W_0 = p_0,\ W_1 = W_0 I\{V_1 \leq W_0\} + V_1 I\{V_1 > W_0\}$ and in general $W_n = W_{n-1} I\{V_n \leq W_{n-1}\} + V_n I\{V_n > W_{n-1}\},\ n \geq 1$. Here, $I\{A\}$ denotes the r.v. which is 1 if the event A occurs, and 0 if it does not. So, for example, $W_1 = W_0$ if $V_1 \leq W_0$ and $W_1 = V_1$ if $V_1 > W_0$. W_n thus denotes the n^{th} renegotiated wage.

The objective is to show that successive increments $\Delta_n = W_n - W_{n-1}$ is negatively correlated for any given F and $\{p_n\}$. To be precise, consider the sign of $Cov(\Delta_n, \Delta_{n+1}) \stackrel{\text{def}}{=} E(\Delta_n \Delta_{n+1}) - E(\Delta_n)E(\Delta_{n+1})$. In particular, consider $Cov(\Delta_1, \Delta_2) = Cov(W_1 - p_0, W_2 - W_1)$, and note that by conditioning on $W_1 = w$ one can equivalently consider $Cov(W_1 - p_0, E(W_2 - W_1 | W_1))$. For negative correlation, it thus suffices to show that $E(W_2 - W_1 | W_1)$ is decreasing in W_1. But $W_2 - W_1$ is conditionally independent of W_1 given W_1, and only depends on a random draw X of F, so it is necessary to only consider showing for $V = (X | X \leq b)$ that the overshoot $E(V - a; V > a) = E(V - a \mid V > a)P(V > a)$, is a decreasing function of a for $b = p_2$. Since $P(V \leq x) = P(X \leq x)/P(X \leq b),\ x \leq b$, and b is constant throughout our analysis here, we can equivalently use X and consider the non-normalized version of the overshoot: $M(a) = E(X - a; a < X < b) = E(X - a \mid a < X < b)P(a < X < b)$.

PROPOSITION. $M'(a) < 0$

Proof. Compute the overshoot as:

$$\begin{aligned} M(a) &= \int_0^\infty P(X - a > x, a < X < b)dx \\ &= \int_a^\infty P(X > x, a < X < b)dx \\ &= \int_a^\infty P(x < X < b)dx \\ &= \int_a^b P(x < X < b)dx. \end{aligned}$$

Hence the derivative of $M(a)$ is given by: $M'(a) = -P(a < X < b) < 0$, as was to be shown.

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Figure 1: Basic Model

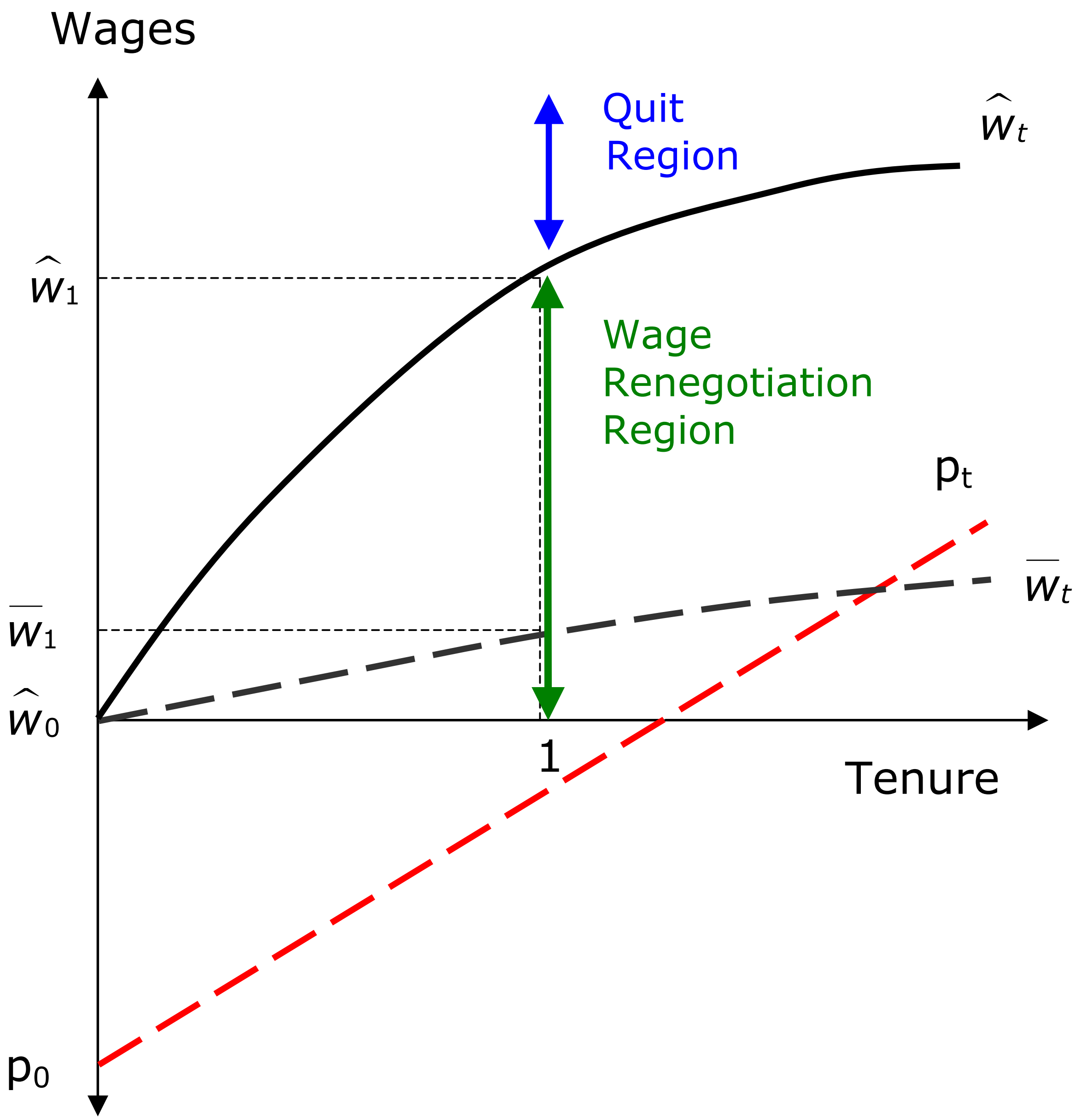

Figure 2: High versus low growth jobs

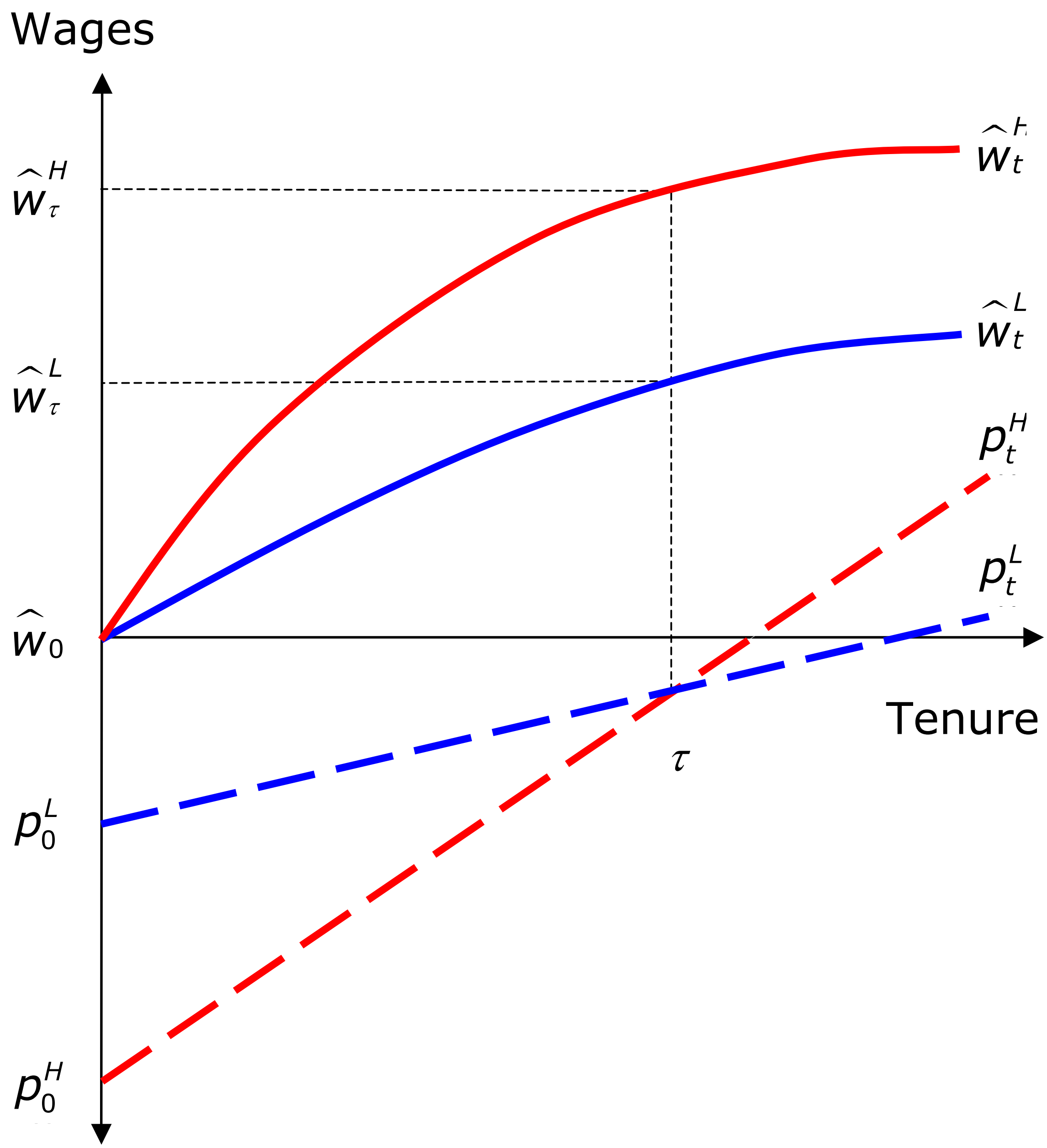

Figure 3: Negative covariance of successive wage increases, conditional on productivity profile

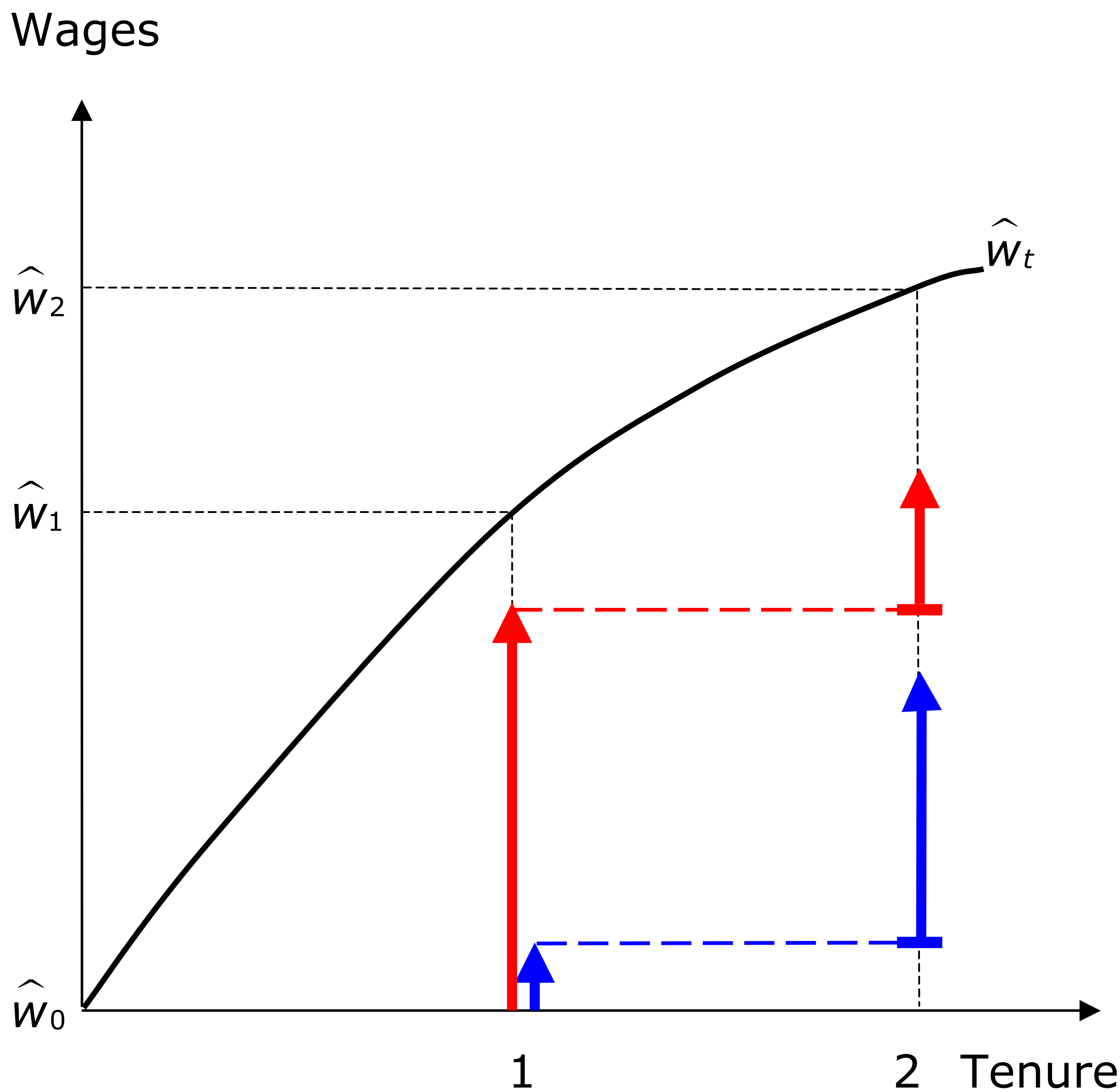

Figure 4: Indeterminacy of unconditional covariance of successive wage increases